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WOOLWICH MATHEMATICAL PAPERS

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WOOLWICH - MATHEMATICAL PAPERS

FOR ADMISSION INTO

THE ROYAL MILITARY ACADEMY

FOR THE YEARS

1891—1900

Eldred John EDITED BY
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MATHEMATICAL EXAMINATION PAPERS

FOR ADMISSION INTO

Royal Military Academy, Woolwich,

JUNE, 1891.

OBLIGATORY EXAMINATION.

I. EUCLID (BOOKS I.—IV. AND VI.).

[*Ordinary abbreviations may be employed; but the method of proof must be geometrical. Proofs other than Euclid's must not violate Euclid's sequence of propositions. Great importance will be attached to accuracy.*]

1. Draw a straight line perpendicular to a given straight line of an unlimited length, from a given point without it.

2. Describe a parallelogram which shall be equal to a given triangle, and have one of its angles equal to a given rectilineal angle.

3. On the perpendicular AD of an equilateral triangle ABC another equilateral triangle EAD is described; show that its perpendicular EF is one-fourth of the perimeter of the triangle ABC.

4. Enunciate that proposition in Euclid's second book which is expressed directly in algebraic symbols by the formula $(2a+b)b+a^2=(a+b)^2$, and give the construction by which the proposition is proved.

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5. In any triangle the square on the side subtending an acute angle is less than the squares on the sides containing that angle by twice the rectangle contained by either of those sides, and the straight line intercepted between the perpendicular let fall on it from the opposite angle and the acute angle.

A, B are two given points, and CD a given straight line; in CD find a point P such that the difference between the squares on AP, BP shall be equal to twice the square on AB.

6. Make a square equal in area to a given equilateral triangle.

7. Draw a tangent to a given circle from a given point outside it.

8. The opposite angles of any quadrilateral figure inscribed in a circle are together equal to two right angles.

If the opposite sides of such a quadrilateral be produced to meet in P and Q, and about the triangles so formed outside the quadrilateral circles be described intersecting again in R, show that P, R, Q will be in one straight line.

9. Describe a circle which shall touch one side of a given triangle and the other two sides produced.

10. Describe an isosceles triangle having each angle at the base double of the vertical angle.

Construct a quadrilateral whose angles, taken in order, are in the proportion $1:2:3:4$, and whose longest side is double of that opposite to it.

11. If the vertical angle of a triangle be bisected by a straight line which cuts the base, the segments of the base shall have the same ratio which the other sides of the triangle have to one another.

The angle A of a triangle ABC is bisected by AD, which cuts the base at D, and O is the middle point of BC; show that OD bears the same ratio to OB that the difference of the sides bears to their sum.

12. In any right-angled triangle, any rectilineal figure described on the side subtending the right angle is equal to the similar and similarly-described figures on the sides containing the right angle.

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II. ALGEBRA.

(Up to and including the Binomial Theorem, the theory and use of Logarithms.)

[N.B.—*Great importance will be attached to accuracy.*]

1. Prove that

$$(4x-1)^3+(2x-3)^3+6(4x-1)(2x-3)(3x-2)=8(3x-2)^3;$$

and divide

$$(x-1)^8+(x-1)^4+1 \text{ by } (x-1)^4+(x-1)^2+1.$$

2. Simplify

$$\frac{2(x^2-1)^2}{(x^2+2)(x^2-16)}-\frac{(x-4)(x-1)^2+(x+4)(x+1)^2}{2x(x^2-16)};$$

and resolve into factors each of the expressions

$$x^2-26x+168 \text{ and } x^4+x^2y^2+y^4.$$

3. Show that the sum of the squares of 5 consecutive numbers is divisible by 5.

4. Find the highest common factor of

$$4x^4+7x^2+16 \text{ and } 4x^4+12x^3+9x^2-16.$$

5. Solve the equations

$$\text{(i.)}\quad \frac{3}{x}+\frac{5}{x^2}=\frac{3}{x-1}+\frac{2}{(x-1)^2};$$

$$\text{(ii.)}\quad \begin{cases}\dfrac{16x^2}{9}-\dfrac{36y^2}{25}=3,\\[2ex] \dfrac{4x}{3}-\dfrac{6y}{5}=1.\end{cases}$$

6. Transform 5363433 from the septenary scale to the denary. Multiply together the two numbers 4535 and 246 in the septenary scale.

7 Solve the equations

$$\text{(i.)}\quad \frac{ax}{b}-\frac{b}{ax}=b-\frac{1}{b};$$

$$\text{(ii.)}\quad \begin{cases}x^3-xy+y^3=4,\\ x+y=1.\end{cases}$$

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8. A courtyard, containing 1300 square feet, is paved with 50 stones of one size and 195 of another. If 10 stones of the first size and 7 of the second make together 100 square feet, find the size of each stone.

9. By *Mathematical Induction*,

(i.) prove the formula for the summation of a Geometrical Progression;

(ii.) show that the series

$$(1^5+1^7)+(2^5+2^7)+(3^5+3^7)+\text{etc. (to } n \text{ terms)} = \frac{n^4(n+1)^4}{8}.$$

10. Find an infinite geometrical progression whose first term is 1, and in which each term is twice the sum of all the terms that follow it.

Define *Harmonical Progression*; and place two terms at each end of the H.P. 15, 20, 30.

11. Find for what value of r the number of combinations of $2m$ things taken r at a time is greatest.

Show that there are two terms in the expansion of $(1-x)^{-\frac{5}{3}}$ greater than all the others, when $x = \frac{15}{17}$.

12. When the logarithms of two adjacent numbers are known, explain how to determine the logarithm of a number lying between them.

Find the tenth root of 4325626.

If £35627. 19*s*. 4*d*. be placed at compound interest at the rate of $3\frac{1}{2}$ per cent. per annum, what will be the amount at the end of 30 years?

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III. PLANE TRIGONOMETRY AND MENSURATION.

(Including the Solution of Triangles.)

[N.B.—*Great importance will be attached to accuracy.*]

1. The tangent of an angle is 2·4. Find the values of the cosecant of the angle, the cosecant of the complement of half the angle, and the cosecant of the supplement of double the angle.

2. Establish the identities—

(i.) $(\operatorname{cosec} A + \sec A)^2 - (\cos A + \sin A)^2$
$$= (\cot A \operatorname{cosec} A + \tan A \sec A)(\sin A + \cos A)(1 + \sin A \cos A).$$

(ii.) $$2\frac{\sin^2 A}{1+\cot A} + \sin 2A + \frac{2\cos^2 A}{1+\tan A} = 2.$$

3. Simplify
$$\frac{\cos(n-2)A - \cos nA}{\sin nA - \sin(n-2)A};$$
and prove that
$$4\tan^{-1}\tfrac{1}{5} - \tan^{-1}\tfrac{1}{239} = \frac{\pi}{4}.$$

4. Prove that
$$\sqrt{1+\tan\theta+\tan^2\theta}\,\sqrt{1+\cot\theta+\cot^2\theta} - \sec\theta\operatorname{cosec}\theta = 1.$$

5. In a plane triangle show that the sines of the angles are to one another as the opposite sides.

Show that
$$a\cos A\cos 2A\sin 4(B-C) + b\cos B\cos 2B\sin 4(C-A)$$
$$+ c\cos C\cos 2C\sin 4(A-B) = 0.$$

6. In the ambiguous case in the solution of plane triangles, if a and b are the two given sides $a < b$, A the given angle, and C_1, C_2 the two values of the angles opposite the third side, prove that
$$\cos A\cos B = \sin\frac{C_1+C_2}{2}\,.\,\sin\frac{C_1-C_2}{2},$$
taking the smaller of the two values of B.

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7. In any triangle ABC, if r, r_a, r_b, r_c, are the radii of the inscribed and of the three escribed circles respectively, and p_a, p_b, p_c, the perpendiculars from the angles on the sides a, b, c, prove that

$$\frac{1}{r_a}+\frac{1}{r_b}+\frac{1}{r_c}=\frac{1}{r}=\frac{1}{p_a}+\frac{1}{p_b}+\frac{1}{p_c};$$

and if the points where the perpendiculars meet the sides be joined, the perimeter of the triangle so formed is $2a \sin B \sin C$.

8. Find the values of the angle C in the triangle ABC from the following data:

$$b = 785, \quad c = 1083, \quad B = 37^\circ\ 11'\ 45''.$$

9. Adapt to logarithmic computation the expression

$$(x+\sqrt{x^2-1})^{\frac{1}{2}}+(x-\sqrt{x^2-1})^{\frac{1}{2}}.$$

10. A railway tunnel consists of a hollow semi-cylindrical top, terminated below in a trough with slanting sides and flat base. The radius of the former being 12 feet, the base and height of the latter being 20 and 18 feet respectively, and the length of the tunnel 1200 yards, find the cost of facing the sides and roof with brick at 1*s.* 6*d.* per square foot ($\pi = \frac{22}{7}$).

11. Within a hollow sphere of 1 foot radius is placed a right prism, the ends of which are equilateral triangles. The side of one of these being 1 foot in length, and the surface of the sphere being in contact with all the six angular points of the prism, find in cubic inches the volume of the latter.

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IV. STATICS AND DYNAMICS.

1. Enunciate and prove the "Triangle of Forces."

A small heavy ring, which can slide freely upon a smooth thin rod, is attached to the end of the rod by a fine string. If the rod be held in any position inclined to the vertical, draw a triangle representing the forces acting upon the ring.

2. A rectangular box, containing a ball of weight W, stands on a horizontal table, and is tilted about one of its lower edges through an angle of 30°. Find the pressures between the ball and the box.

3. State the conditions of equilibrium of three parallel forces acting upon a rigid body.

A rod ABC, 16 inches long, rests in a horizontal position upon two supports at A and B one foot apart, and it is found that the least upward and downward forces applied at C which would move the rod are 4 oz. and 5 oz. respectively. Find the weight of the rod and the position of its centre of gravity.

4. Show how to find the centre of gravity of a body composed of two parts whose weights and centres of gravity are known.

A solid figure is formed of an upright triangular prism surmounted by a pyramid; if the length of every edge of this figure be a feet, find the height of its centre of gravity above the base.

5. Show that the algebraical sum of the moments of two forces (whose lines of action intersect) about any point in the plane containing the forces is equal to the moment of their resultant.

OA, OB are chords, 4 and 5 inches in length, of a circular disk $OACB$, whose diameter OC is 6 inches. If forces of 3 and 4 lbs. act from O along these chords respectively, find how the disk will begin to move, the point C being fixed.

6. Describe, and show how to graduate, the Danish Steelyard.

7. Explain how the measure of the velocity of a moving point depends upon the units of space and time.

If the unit of time be half a minute and the unit of length be 2 yards, what will be the measure of the velocity of a body which describes, at a uniform rate, 14 miles in 3 hours?

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8. Investigate the formula $s = \frac{1}{2}ft^2$ for the space described from rest by a particle subject to uniform acceleration f, and hence deduce a corresponding expression in the case where the particle has an initial velocity u.

A particle is observed to describe 7 feet in 3 seconds, and 13 feet in the next 3 seconds; find its acceleration.

9. The side BC of a triangle ABC is vertical; show that, if the times of falling down the two sides BA, AC be equal, the triangle must be isosceles or right angled.

10. Distinguish, with examples, between the *volume*, *mass*, and *weight* of a body, and find the relation between the units of mass and weight in order that W may be equal to Mg.

If the measures of the mass and weight be the same, and the unit of length be 2 feet, find the unit of time.

11. If a force of 15 poundals act upon a mass of 13 pounds, what velocity will it generate in 8 seconds?

12. A particle is projected in any manner in a vertical plane, show how to find its position at the end of a given time.

A stone is thrown from the top of a tower with a velocity of 50 feet a second in a direction making an angle of 30° with the horizon. Find the distance of the stone from the point of projection at the end of 5 seconds.

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FURTHER EXAMINATION.

V. MATHEMATICS (I).

[*Full marks may be obtained for about* three-fourths *of this paper. Great importance is attached to accuracy.*]

1. Define a "conic section," and show that a straight line *generally* cuts a conic section in two points.

Enumerate carefully all the cases of exception.

2. Prove that the subtangent at any point of a parabola is double of the abscissa, and the subnormal is half of the latus rectum.

If one side of an equilateral triangle be a focal radius SP of a parabola, and another side lie along the axis, show that the third side will be either the tangent or the normal at P, and find in each case the length of SP.

3. Prove that, in any central conic, the square on the ordinate PN bears a constant ratio to the rectangle under the abscissae AN, $A'N$.

PNP' is a double ordinate of an equilateral hyperbola whose transverse axis is AA'. Prove that the directions of AP, $A'P'$ are perpendicular to one another.

4. If from any point O two straight lines be drawn, OPQ, $OP'Q'$, intersecting an ellipse in P, Q; P', Q', show that the rectangles $OP \cdot OQ$, $OP' \cdot OQ'$ are to one another as the squares of the diameters parallel to PQ, $P'Q'$ respectively.

If an ellipse, inscribed in a triangle, touch one side in its middle point, prove that the straight line joining that point with the opposite angle of the triangle will pass through the centre of the ellipse.

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5. If from any point P on a hyperbola a straight line, drawn in a given direction, meet the two asymptotes in R, R' respectively, prove that the rectangle $RP \cdot PR'$ is constant.

Hence show that a straight line touching the hyperbola, and terminated by the asymptotes, is bisected at the point of contact.

6. If a right cone be cut by a plane, determine by geometrical construction the positions of the foci and directrices of the curve of section.

Show that sections made by parallel planes have the same eccentricity.

7. Given the coordinates of two points, P, Q, find the coordinates of the point in which the straight line PQ is divided in a given ratio.

Apply your results to show that the straight line joining the middle points of the diagonals of a quadrilateral, and the straight line joining the middle points of either pair of opposite sides, bisect each other.

8. Find the angle between the straight lines whose equations, referred to rectangular axes, are

$$lx + my + c = 0, \quad l'x + m'y + c' = 0.$$

$OACB$ is a parallelogram, whose sides OA, OB are of lengths a, b, and make angles α, β respectively with the axis Ox. Write down the equations of the two diagonals OC, AB, and show that the tangent of the angle between them is

$$\frac{2ab \sin(\beta - \alpha)}{a^2 - b^2}.$$

9. Prove that the equation of any tangent to the circle $x^2 + y^2 = c^2$ may be written in the form

$$x \cos \alpha + y \sin \alpha = c.$$

Prove also that the locus of the foot of the perpendicular let fall upon the tangent, from the point in which the circle cuts the positive direction of the axis of y, is the curve whose equation is

$$(x^2 + y^2 - yc)^2 = c^2\{x^2 + (y - c)^2\}.$$

10. Show that the equation of the normal to the parabola $y^2 = 4ax$, which is inclined at an angle θ to the axis, may be written in the form

$$y = (x - a - a \sec^2\theta) \tan \theta.$$

If a chord of a parabola, whose inclination to the axis is $\tan^{-1}\sqrt{2}$, be normal to the curve at one end, show that it will subtend a right angle at the vertex.

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11. Investigate the equation of the polar of any point with respect to a given ellipse or hyperbola.

An ellipse and a hyperbola have the same principal axes. Show that the polar of a point on either curve with respect to the other touches the first curve.

12. Write down the equations of the latera recta of the conic whose polar equation is

$$\frac{L}{r} = 1 - e\cos\theta.$$

Also show that if r_1, r_2, r_3 be the lengths of three focal radii, each of which is equally inclined to the other two,

$$\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}=\frac{3}{L}.$$

VI. MATHEMATICS (2).

[*Full marks may be obtained for about* three-fourths *of this paper. Great importance is attached to accuracy.*]

1. If $a^3+b^3+c^3 = 3abc$, where a, b, c are finite, show that any one of the three quantities

$$a^2-bc, \quad b^2-ca, \quad c^2-ab,$$

is a mean proportional between the other two.

2. Find the coefficient of x^6 in the expansion of

$$(1+x^2+x^4)(2-3x-2x^2)^{-1}$$

in a series of powers of x, and show that

$$\sqrt[6]{\cdot 875} = \cdot 97365 \text{ nearly.}$$

3. A bag contains 3 balls, which are equally likely to be either white, black, or red. A white ball is drawn and replaced, and then a black ball is drawn and replaced. Find the probability that the next drawing will give a red ball.

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4. Express

$$\sqrt{a^2+\frac{2a}{2a+1}}$$

as a continued fraction, and show that

$$1+\frac{1}{2+}\,\frac{1}{6+}\,\frac{1}{2+}\,\frac{1}{6+}\cdots$$

is double of

$$\frac{1}{1+}\,\frac{1}{2+}\,\frac{1}{1+}\,\frac{1}{2+}\cdots.$$

5. In how many different ways can 3 white, 3 red, and 3 blue counters be arranged in 3 rows of 3 each?

If there were n counters of each of n different colours, show that the number of ways in which the n^2 counters could be arranged in n rows of n each is

$$\frac{\lfloor n^2}{1^n \times 2^n \times 3^n \times \ldots \times n^n}.$$

6. Solve the simultaneous equations

$$\text{(i.)}\quad \begin{cases} x^2+y^2 = 2a^2 \\ x^3-y^3 = 2a^3. \end{cases}$$

$$\text{(ii.)}\quad \begin{cases} (x\cos\alpha - c)\sin(\alpha+\beta) = (y\cos\alpha - c)\sin(\alpha-\beta) \\ x\cos(\alpha+\beta) = y\cos(\alpha-\beta). \end{cases}$$

Show that in (ii.), $x+y = 2c\cos\alpha$.

7. If θ be the circular measure of an angle less than a right angle, prove that

$$\sin\theta > \theta - \frac{\theta^3}{4}, \text{ and } \tan\theta > \theta + \frac{\theta^3}{4}.$$

8. A man, travelling due north along a straight road, observes the points A, B at which two distant objects P, Q lie respectively due east, and at an intermediate point C he measures the distances AC, BC, and the angles PCA, QCB. Show that he will have data from which he can calculate the length of PQ and its inclination to the direction of the road.

Show that this inclination will be 30°, if each of the measured angles is 60°, and one of the measured distances is double of the other.

9. If $2\cos\theta = x+\frac{1}{x}$, express $\cos n\theta$ and $\sin n\theta$ in terms of x.

Also prove that

$$\frac{\tan^{-1}(\sqrt{-1}\sin\theta)}{\sqrt{-1}} = \log\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right).$$

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10. An open box in the shape of a perfect cube lies on a horizontal plane with the diagonals of its base pointing towards the four cardinal points. Show that at noon, when the sun's altitude is $\tan^{-1}(2\sqrt{2})$, the portions of the interior surface in sunlight and in shade are as 37 : 43.

11. Find the sum of each of the infinite series

(i.) $\frac{1}{2.3}+\frac{1}{4.5}+\frac{1}{6.7}+\dots;$

(ii.) $\frac{1}{3.5}+\frac{1}{7.9}+\frac{1}{11.13}+\dots;$

(iii.) $\frac{1}{\lfloor 2}-\frac{1}{\lfloor 4}+\frac{1}{\lfloor 6}-\dots.$

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VII. MECHANICS.

[*Full marks may be obtained for about* three-fourths *of the paper. Mathematical instruments must be used for questions requiring a graphical method of solution.*]

1. Solve graphically the following problem:—A uniform beam AB, weight 100 lbs., is supported by two strings AC, BD, the former making an angle of 51° with the horizontal, and the latter being vertical. It is maintained at an angle of 5° to the horizon by a horizontal force P applied at B. Find, by graphical construction, the value of P in pounds.

2. Solve the following problem:—A uniform horizontal beam AB, weight 100 lbs., is placed with the end A resting against a rough wall AD, and is supported by a string CD. Find, by construction, the tension of CD in pounds, and the coefficient of friction in operation between the beam and the wall at A.

$$AB = 4', \quad AC = 3', \quad AD = 2'.9''.$$

3. Three forces, P, Q, R, make equilibrium; P is given in magnitude and position, Q in magnitude only, R in direction only, making an angle θ with P. Determine Q and R either by construction or by calculation. Show that there are generally two solutions, and that if $P > Q$ there are limits to the angle θ, beyond which the question is impossible of solution.

4. If four forces in equilibrium act along the sides of a parallelogram, show that they are as the sides in which they act.

If four forces in equilibrium act at right angles to the sides of a parallelogram (not rectangular) at their middle points, prove that they are as the sides on which they act.

5. A rectangular block $ABCD$, whose height is double its base, stands with its base AD on a rough floor, coefficient of friction $\frac{1}{8}$. If it be pulled

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by a horizontal force at C till motion ensues, determine whether it will slip on the floor, or begin to turn over round D.

6. The masses of two particles at A, B are m, m': if P be any point, prove that the resultant of two forces represented by $m \,.\, AP$, $m' \,.\, BP$ acts in GP, and is represented by $(m+m')GP$, where G is the centre of gravity of m, m'. Hence show that if G is the centre of gravity of the masses m, m', m'' at any three points A, B, C, the three forces

$$m \,.\, AG, \quad m' \,.\, BG, \quad m'' \,.\, CG$$

make equilibrium.

If OA, OB, OC are three diverging bars of the same material and same section, prove that if their centre of gravity is at O, the sines of the angles they make are as the squares of their lengths.

7. Find the slope of a smooth inclined plane, if the work done in drawing a heavy body up a given length of it, is equal to that done in drawing the body along an equal length of a rough floor (coefficient of friction $\frac{1}{2}$).

If the floor and plane are equally rough ($\mu < 1$), prove that more work is done in drawing it up the plane, than along an equal length of the floor, whatever be the slope.

8. If a body is projected vertically upwards, prove that the velocity at any point in its ascent is equal to its velocity when passing through that point in its fall.

9. Find the amount of *vis viva* lost in the direct collision of two inelastic balls, masses M, m, velocities u, v.

If the balls are equal and going in the same direction, show that less than half their *vis viva* is lost.

10. A flexible heavy string, length $2l$, is moving over a smooth fixed small pulley, the two unequal portions of it hanging vertically. Prove that at the instant when its middle point is at a distance x below the pulley, the acceleration with which it is moving is

$$f = \frac{x}{l} g.$$

Find also the tension of the string at any assigned point of the descending portion at the same instant.

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11. A projectile thrown at a small elevation (3°), gives a range of 1000 yards on a horizontal plane. If the plane, instead of being horizontal, had an upward slope of 1°, what would be the range in yards, approximately?

12. A heavy bead, loosely strung on a smooth vertical circular wire, falls down it from rest at the highest point O. When at any assigned point, find the velocity with which its distance from O (in a straight line), is increasing.

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MATHEMATICAL EXAMINATION PAPERS

FOR ADMISSION INTO

Royal Military Academy, Woolwich,

NOVEMBER, 1891.

OBLIGATORY EXAMINATION.

I. EUCLID (BOOKS I.—IV. AND VI.).

[*Ordinary abbreviations may be employed, but the method of proof must be geometrical. Proofs other than Euclid's must not violate Euclid's sequence of propositions. Great importance will be attached to accuracy.*]

1. Draw a straight line at right angles to a given straight line, from a given point in the same; and show that two straight lines cannot have a common segment.

2. Give Euclid's Axiom on parallel straight lines; and show that the straight lines which join the extremities of two equal and parallel straight lines towards the same parts are also themselves equal and parallel.

Two quadrilaterals $ABCD$, $EFGH$, have the sides AB, DC respectively equal to the sides EF, HG; and also the angles which the diagonals AC, BD make with AB, CD, namely, CAB, ABD, BDC, DCA respectively equal to the angles which the diagonals EG, FH make with EF, HG, namely, GEF, EFH, FHG, HGE; show that the quadrilaterals are equal in every respect.

3. To a given straight line apply a parallelogram, which shall be equal to a given triangle, and have one of its angles equal to a given rectilineal angle.

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4. Enunciate the two propositions represented by

$$(2a \pm b)^2 + b^2 = 2\{a^2 + (a \pm b)^2\}.$$

AB is bisected at C and produced to D; AE, which is at right angles to AB, is divided into two equal parts at F and into two unequal parts at G; CH, BI, DKL are drawn parallel to AE; and FK, GHL, EI parallel to AD. Show that the sum of the squares on GD and IL is double of the sum of the squares on FC and HK.

5. In obtuse-angled triangles, if a perpendicular be drawn from either of the acute angles to the opposite side produced, the square on the side subtending the obtuse angle is greater than the squares on the sides containing the obtuse angle, by twice the rectangle contained by the side on which, when produced, the perpendicular falls, and the straight line intercepted without the triangle, between the perpendicular and the obtuse angle.

6. From any point in the diameter of a circle which is not the centre there can be drawn to the circumference two straight lines, and only two, which are equal to one another, one on each side of the diameter.

If two such pairs of equal straight lines are drawn, prove that the chords joining the extremities of the unequal straight lines meet upon the diameter of the circle (produced if necessary).

7. If from any point without a circle two straight lines be drawn, one of which cuts the circle, and the other touches it; the rectangle contained by the whole line which cuts the circle, and the part of it without the circle, shall be equal to the square on the line which touches it.

8. A quadrilateral $ABCD$ is circumscribed round a circle, touching the circle in E, F, G, H. Find the difference of two opposite angles of $ABCD$ in terms of the difference of two adjacent angles of $EFGH$.

9. Describe a circle about a given triangle.

10. Inscribe a regular quindecagon in a given circle.

11. The sides about the equal angles of equiangular triangles are proportionals.

If in two triangles ABC, DEF, AB is to BC as DE is to EF, and the angle A is equal to the angle D, but the angle B unequal to the angle E; show that the angles C and F are supplementary.

12. In equal circles angles, whether at the centres or at the circumferences, have the same ratio which the arcs on which they stand have to one another.

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II. ALGEBRA.

(Up to and including the Binomial Theorem, the theory and use of Logarithms.)

[N.B.—*Great importance will be attached to accuracy.*]

1. Find the value of

$$1680+(r-7)[1470+(r-8)\{378+(r-9)(35+r-10)\}]$$

when $r=-7$.

2. Multiply

$$2x^5+4x^4+6x^3+8x^2+10x+5 \text{ by } x^2-2x+1.$$

Find an algebraic function such that when it is divided by a^2-ab+b^2, the quotient is $3a^2-2ab+b^2$ and the remainder is $2a^3(b-a)$.

3. Write down one factor of $x^{17}-17x^2+16$.

Divide $x^{17}-17x^2+16$ by x^2-2x+1, so far as to show the four terms of highest degree in the quotient.

4. Define the algebraical Greatest Common Measure and Least Common Multiple of two or more functions.

Find the Greatest Common Measure of

$$7x^4+6x^3-8x^2-6x+1 \text{ and } 11x^4+15x^3-2x^2-5x+1.$$

5. Simplify the expression

$$\frac{(x-y)^3+(z-y)^3-(x-2y+z)^3}{(x-y)(z-y)(x-2y+z)}.$$

6. Solve the equations

$$\text{(i.)}\quad \frac{1}{x+2}+\frac{1}{x+3}-\frac{3(x+6)}{(x+2)(x+3)}=6-\frac{6x+23}{x+3};$$

$$\text{(ii.)}\quad 11x+4y=71,$$
$$11y+4x=51.$$

7. Examine the meaning of $a^{\frac{1}{2}}$ and a^0 in the theory of indices.

If $$3c=\sqrt[3]{100}+2\sqrt[3]{10}+3,$$
show that $$3(c-1)^3-20c=0.$$

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8. Solve the equations

$$\text{(i.)}\quad x^2 - 18x - 72 = 0;$$

$$\text{(ii.)}\quad \sqrt{3}\sqrt{(x^4 + 2x^2 + 3)} + \tfrac{1}{3}x^2 + 3 = \tfrac{2}{3}ax^2.$$

9. Find the sum of 19 terms of the Arithmetical Progression whose 3rd and 13th terms are 17 and 87 respectively.

If $x!$ denote the product of the first x integers, show that

$$1 \,.\, 1! + 2 \,.\, 2! + 3 \,.\, 3! + \ldots + x \,.\, x! = (x+1)! - 1.$$

10. Find the number of permutations of n different things taken 4 together.

If there are 4 flags of each of 6 different colours, prove that 4 flags may be hoisted into a vertical line in 6^4 different ways.

11. Expand $\dfrac{1}{(1-x)^n}$ by the binomial theorem, stating the necessary imitations on the value of x.

If

$$(1-x)^{-1}(1-x^2)^{-p}(1-x^3)^{-q}(1-x^4)^{-r} = 1 + x + 2(px^2 + qx^3 + rx^4) + \ldots$$

show that $$p = \tfrac{1}{2}q = \tfrac{1}{6}r = 1.$$

12. If a number has no integral part, show that the characteristic of its logarithm to base 10 is negative, and numerically exceeds by unity the number of zeros following the decimal point.

Calculate $$(2{\cdot}718)^{3{\cdot}142} \times (3{\cdot}142)^{2{\cdot}718}$$

by the aid of a table of logarithms.

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III. PLANE TRIGONOMETRY AND MENSURATION.

(Including the Solution of Triangles.)

[N.B.—*Great importance will be attached to accuracy.*]

1. Trace the changes in sign and magnitude

$$(1) \text{ of } \sin\theta\,; \quad (2) \text{ of } \sin\left(\frac{\pi}{6}+\frac{\pi}{3}\sin\theta\right),$$

as θ changes from 0 to 2π.

2. Prove that

(i.) $(\sec^2 A+\tan^2 A)(\operatorname{cosec}^2 A+\cot^2 A) = 1+2\sec^2 A\operatorname{cosec}^2 A.$

(ii.) $\dfrac{\sec A+\tan A}{\operatorname{cosec} A+\cot A}-\dfrac{\sec A-\tan A}{\operatorname{cosec} A-\cot A} = 2(\sec A-\operatorname{cosec} A).$

3. Having given $\sin A = \frac{4}{5}$ and $\sin B = \frac{8}{17}$, find the value of $\tan(A+B)$.

4. Prove that

$$\sin\tfrac{1}{2}(A-B)\{\cos A+\cos B-\sin(A+B)\}$$
$$= \cos\tfrac{1}{2}(A+B)\{\sin(A-B)+\cos A-\cos B\}.$$

5. Find an expression for all the angles which have the same sine as a given angle.

Having given the value of $\sin 2A$, find the number of values of $\tan A$.

6. Prove that

$$\tan^{-1}x+\tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy},$$

and that

$$2\tan^{-1}\tfrac{1}{7}+\tan^{-1}\tfrac{17}{31} = \frac{\pi}{4}.$$

7. *Find* the sine and cosine of 18° and also the sine and cosine of 54°.

8. If R is the radius of the circumscribing circle of a triangle ABC, prove that

$$\frac{\cos A}{c\sin B}+\frac{\cos B}{a\sin C}+\frac{\cos C}{b\sin A} = \frac{1}{R}.$$

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9. Write down the expression for the cosine of an angle of a triangle in terms of its sides, and prove that, for a triangle ABC,

$$\sin\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}.$$

Having given that the sides of a triangle are 35·3, 34·5, and 1·2 find its greatest angle.

10. An observer standing at the distance of 52 feet from the foot of a tower finds that the angle of elevation of the top of the tower is 42° 15′ 15″; find the height of the tower, the observer's eye being 5 feet from the ground.

11. A piece of wood is in the form of a regular pyramid on a square base; the side of the base is 6 inches, and the perpendicular distance of the vertex from the base is 8 inches; find the number of cubic inches in the volume of the wood, and the number of square inches in its surface.

12. A cylindrical boiler is hemispherical at its two ends; its radius is 2 feet, and its total length is 8 feet; assuming that a cubic foot of water weighs 62·5 lbs., find the number of tons of water which will fill the boiler. (Take $\pi = 3·14$.)

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IV. STATICS AND DYNAMICS.

[*The acceleration due to gravity may be taken as equal to* 32.]

1. State and prove the proposition known as the "Polygon of Forces."

Forces 1, 2, 3 and $2\sqrt{2}$ act on a point in the directions of the sides AB, BC, CD and the diagonal DB of a square $ABCD$ respectively; determine their resultant, graphically or otherwise.

2. Each of a pair of sculls has four-fifths of its length outside the rowlock, and a man sculling pulls at the handle of each with the force P. Another man thrusts an oar over the stern against the bottom of the water with the force $2P$ at an angle of 60° to the horizon. Compare their effects in propelling the boat.

3. State the conditions of equilibrium of any number of forces acting on a body in one plane.

A smooth wall is inclined at an angle of 60° to the horizon; a heavy uniform rod AB $4\sqrt{6}$ feet long, is in equilibrium at an angle of 45° to the wall, its lower end, A, rests on the wall, and a point in it, C, rests on a smooth horizontal rail parallel to the wall. Draw a diagram showing how the forces act, and find the distance of C from the wall.

4. Find the centre of mass of a uniform triangular board.

The sides AB, BC, CD, DA, of a trapezium are of lengths 54, 36, 27, 45 respectively, AB being parallel to CD. Prove that its centre of gravity is at a distance 16 from AB.

5. Find the ratio of P to W in a smooth screw.

If a power of 1 cwt. acting horizontally and at right angles to the extremity of an arm 8 ft. 4 in. long will raise 5 tons by means of a screw whose axis is vertical and diameter 2 inches, find the inclination to the horizon of the thread of the screw.

6. Find the relation between the power and weight in that system of pulleys in which each string is attached to the weight, the strings being parallel and the weights of the pulleys being neglected.

If there be three pulleys, two of them moveable, and the weight of each $\frac{1}{4}$ lb., what will be the value of P when $W = 8$ lbs.?

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7. What is meant by uniform acceleration?

A stone is thrown vertically upwards, with a velocity 36 feet per second. After what times will its velocity be 12 feet a second?

8. A particle moving in a straight line with a uniform acceleration a passes a certain point A with velocity u. State (without proof) the formulae which give (1) the distance s subsequently moved over in time t, and (2) the velocity v with which the particle will be moving when at a distance d from A.

A railway train, moving with velocity 48 miles an hour, has its velocity reduced to 16 miles an hour in 5 minutes. Find the space passed over in the interval, the retardation being assumed to be uniform.

9. Two smooth inclined planes have a common altitude, and their inclinations are 30° and 60° to the horizon. Two masses start simultaneously from the common vertex to fall one down each plane. Compare (1) their times of falling to the bottom, and (2) their final velocities.

10. A particle is projected horizontally from the top of a vertical wall 16 feet high with a velocity $32\sqrt{3}$ feet per second. Find its range on the horizontal ground, and prove that when it strikes the ground its velocity is 64 feet per second.

11. State the law of Newton which tells us how to measure force.

If the unit of force were that which acting on 1 ton of mass would in 1 minute generate a velocity of 1 mile a minute, how many units of force would the weight of 1 ton contain?

A man 10 stone in weight, who pulls with a force equal to his own weight, drags a 10 ton railway carriage from rest on a smooth horizontal line of rails. How far will he move it in one minute?

12. A mass (m), falling vertically, draws a mass M along a smooth horizontal table by means of a fine string which passes over a smooth pulley at the edge of the table; find the tension of the string at any time and the acceleration produced. If the tension is equal to half the weight of m what is the ratio of M to m?

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FURTHER EXAMINATION.

V. MATHEMATICS (1).

[*Full marks may be obtained for about* four-fifths *of this paper. Great importance is attached to accuracy.*]

1. Explain the meanings of a, b, α, p, m, c in the following equations representing a straight line:

$$\frac{x}{a}+\frac{y}{b}=1, \quad x\cos\alpha+y\sin\alpha-p=0, \quad y=mx+c.$$

What are the relations between these constants, if all three equations represent the *same* straight line?

2. Find the equations of the sides of a triangle, the coordinates of whose vertices are (1, 4), (2, −3), (−1, −2), respectively.

Also find the equations of the bisectors of the angles between the lines

$$12x+5y-4=0, \quad 3x+4y+7=0.$$

3. Find the locus of a point P, so that when PM and PN are drawn respectively perpendicular to two fixed lines, OM and ON, the sum of OM and ON is given.

4. Find the locus of a point P (1) when the tangents from P to two given circles are in a given ratio, (2) when the sum of the tangents is constant.

5. Being given the focus and two points on a parabola, find the position of its directrix, and state the number of solutions of the problem.

6. Prove that the normal at any point on an ellipse bisects the angle between the focal vectors of the point.

Draw the normals that pass through a given point on the axis minor of an ellipse, and find when a solution (other than the minor axis itself) is impossible.

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7. Show that an ellipse can be projected orthogonally into a circle.

Hence prove that any triangle can be projected orthogonally into an equilateral triangle.

8. In the transformation of the general equation of the second degree from one rectangular system of coordinate axes to another, show that the quantities $a+b$ and $ab-h^2$ remain unaltered, where a, b, $2h$ are the coefficients of x^2, y^2, xy respectively.

Find the lengths of the axes of the conic that is represented by the equation

$$12x^2 - 12xy + 7y^2 = 48.$$

9. Find what curve is represented by the equation

$$\sqrt{\frac{x}{a}} + \sqrt{\frac{y}{b}} = 1,$$

and give a figure of the curve.

10. Prove that the sum of the squares of any pair of conjugate diameters of an ellipse is constant.

Being given in magnitude and position two conjugate diameters of an ellipse, find, by geometrical construction, the position and magnitude of its axes.

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VI. MATHEMATICS (2).

[*Full marks may be obtained for about* three-fourths *of this paper. Great importance is attached to accuracy.*]

1. Solve the equations

$$\text{(i.)}\quad (x+4)(x-3) = 5\sqrt{(x+3)(x-2)}\,;$$

$$\text{(ii.)}\quad \left.\begin{aligned} x^2-2xy+3y^2 &= a^2-2ab+3b^2 \\ 3x^2-2xy+y^2 &= 3a^2-2ab+b^2 \end{aligned}\right\}.$$

2. Find the sum of the series

$$1+3+6+\ldots+\tfrac{1}{2}n(n+1).$$

The top and bottom layers of a pile of spherical shot are equilateral triangles, whose sides contain a and b shot respectively. Show that the number of shot in the pile

$$=\tfrac{1}{6}(b-a+1)\{b^2+b(a+2)+a(a+1)\}.$$

3. Show that

$$\sqrt{x^2+4}-\sqrt{x^2+1} = 1-\frac{1}{4}x^2+\frac{7}{64}x^4-\text{etc.}$$

$$\text{or } = \frac{3}{2x}\left\{1-\frac{5}{4x^2}+\frac{21}{8x^4}-\text{etc.}\right\}$$

4. Show that the expression

$$\frac{1}{b-c}+\frac{1}{c-a}+\frac{1}{a-b}$$

will be positive if a, b, c; b, c, a, or c, a, b are in descending order of magnitude.

5. Explain how to solve in integers an indeterminate equation of the first degree in two variables.

Find a number of two digits such that if it be multiplied by three, and added to four times the number formed by reversing the digits, the sum is 659.

6. If A and B be two acute angles, such that $\sin A = \frac{4}{5}$ and $\sin B = \frac{5}{13}$, prove (without using tables) that $A+2B$ is greater than 90° and less than 180°.

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7. Explain what is meant by Mathematical Induction, and show by means of the identity

$$\cos(n+1)\theta+\cos(n-1)\theta = 2\cos n\theta\cos\theta,$$

that if $2\cos\theta = x+\frac{1}{x}$, then $2\cos n\theta = x^n+\frac{1}{x^n}$, n being a positive integer.

8. A, B are two stations a miles apart, and P a point moving in a straight line. At a given moment the angles BAP, ABP are observed to be α, β, and after a certain interval to be $\alpha+\gamma$, $\beta-\gamma$. Find the distance described by the point in the interval.

9. Investigate expressions for the radii of the inscribed and circumscribed circles of a triangle in terms of the sides of the triangle.

Find their numerical values in the case of a triangle in which

$$a = 36, \quad B = 73^\circ\ 15', \quad C = 45^\circ\ 30'.$$

10. Sum the series

$$\sin\alpha+\sin(\alpha+\beta)+\sin(\alpha+2\beta)+\ \dots \text{ to } n \text{ terms.}$$

The right angle C of a triangle ABC is divided into n equal parts by lines which meet the hypotenuse AB in points, P_1, P_2, P_3, ... P_{n-1}; show that

$$\frac{1}{CP_1}+\frac{1}{CP_2}+\dots+\frac{1}{CP_{n-1}} = \frac{a+b}{2ab}\left\{\cot\frac{\pi}{4n}-1\right\}.$$

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VII. MECHANICS.

[*Full marks may be obtained for about* three-fourths *of the paper. Mathematical instruments must be used for questions requiring a graphical method of solution.*]

1. Enunciate and prove the Polygon of Forces.

Five forces OA, OB, OC, OD, OE acting at the point O are in equilibrium. If the forces in OA, OB, OD be respectively 4, 2, 3, find graphically the forces in OC and OE. Given

$$\angle AOB = 45^\circ, \quad \angle AOC = 120^\circ, \quad \angle AOD = 210^\circ, \quad \angle AOE = 270^\circ.$$

2. A rod ACB weighing 25 ozs. rests upon a smooth peg C, and its end A is attached to a fixed point O in the same horizontal line with C by means of a string OA. Find graphically the position of the centre of gravity of the rod and the magnitudes of the tension of the string and the pressure between the rod and peg.

$$AC = 4\tfrac{1}{4}, \quad AO = 2\tfrac{1}{4}, \quad OC = 2\tfrac{1}{2} \text{ inches.}$$

3. A, B, C are three fixed smooth pegs in a vertical plane, A being 3 feet vertically below B and 4 feet horizontally to the right of C. A string 13 feet long passes round the three pegs and has its extremities attached to a weight W; find the tension of the string and the resultant pressure on each peg.

4. The weights of a system of heavy bodies are W_1, W_2, ... and the coordinates of their centres of gravity are x_1, y_1; x_2, y_2; ... respectively. Find the coordinates of the centre of gravity of the system.

A square board is divided into 36 equal squares by lines drawn parallel to the sides. A figure is formed by taking the 1st and 4th squares in the first column, the 1st, 2nd, and 4th squares in the second column, the 2nd, 3rd,

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4th, and 5th squares in the third column, the 4th, 5th, and 6th squares in the fourth column, the 2nd, 3rd, 4th, 5th, and 6th squares in the fifth column, and the 3rd, 5th, and 6th squares in the last column. Show that the centre of gravity of the figure formed of these 20 squares divides the line AB in the ratio of 11 to 13.

5. A uniform rod rests on the rim of a plate with its middle point at the centre of the rim. The plate itself rests on a horizontal plane. A downward vertical force, just sufficient to disturb equilibrium, being applied to one end of the rod, show that the plate and rod will begin to move together, or the rod only, according as the weight of the rod is greater or less than $\frac{3}{8}$rds of the weight of the plate; $4a$, $5a$ being the diameters of the base and rim of the plate respectively, and $6a$ the length of the rod.

6. Define *work*, and show how it is measured.

A right-angled triangle ABC turns stiffly in its own plane about the middle point of the hypotenuse AB. If forces P, Q, R, just sufficient to overcome the resistance, be applied at right angles to the sides BC, CA, AB at the angular points, all tending to turn the triangle in the same direction, show that the work done by them in turning the triangle through a right angle $= \frac{1}{2}\{Rc + (P - Q)(a - b)\}$; the forces remaining throughout the motion parallel to their original directions and constant in magnitude.

7. A heavy particle slides down a rough inclined plane; find the space described from rest in a given time.

Two particles are projected with a velocity of 40 feet per second from points 88 feet apart, the one up and the other down, a rough plane ($\mu = \frac{1}{4}$) inclined to the horizon at an angle $\tan^{-1}\frac{7}{24}$. Find when and where they will meet, and account for the double solution [$g = 32$].

8. Within a smooth circular tube fixed in a vertical plane are two particles of mass P, Q connected by a string whose length is equal to half that of the tube. Find the acceleration of each particle in the direction of motion, and the tension of the string supposed tight, when the line joining the particles makes an angle θ with the horizon.

9. Two smooth imperfectly elastic balls, moving in one plane with given velocities in given directions, impinge obliquely on each other; determine the motion of each after impact.

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10. A particle hangs from a fixed point in a wall by a string of length a, find the least velocity which must be given to it in order that it may make a complete revolution, without the string becoming slack.

If the string come in contact with a nail in the wall situated in the horizontal line through the point of suspension and at a distance b from it, find the least initial velocity in order that the particle may make a complete revolution round the nail, without the string becoming slack.

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MATHEMATICAL EXAMINATION PAPERS

FOR ADMISSION INTO

Royal Military Academy, Woolwich,

JUNE, 1892.

OBLIGATORY EXAMINATION.

I. EUCLID (BOOKS I.—IV. AND VI.)

[*Ordinary abbreviations may be employed; but the method of proof must be geometrical. Proofs other than Euclid's must not violate Euclid's sequence of propositions. Great importance will be attached to accuracy.*]

1. The sum of any two sides of a triangle is greater than the third side, and their difference is less than the third side.

2. If two quadrilaterals $ABCD$, $EFGH$ have the four angles A, B, C, D respectively equal to the four angles E, F, G, H; and likewise the sides AB, DC respectively equal to the sides EF, HG; show that if AD and BC meet when produced the quadrilaterals are equal in every respect.

3. Define *parallel straight lines*; and show that parallelograms on the same base and between the same parallels are equal to one another.

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4. Enunciate the proposition which is represented in algebraical symbols by

$$(a+b)^2+a^2 = 2a(a+b)+b^2;$$

and give the construction by which the proposition is proved.

If in a quadrilateral $ABCD$, the sides AD and BC are each perpendicular to the side AB, show that the square on DC is less than the sum of the squares on the three other sides by twice the rectangle contained by AD and BC.

5. Divide a given straight line into two parts, so that the rectangle contained by the whole and one of the parts may be equal to the square on the other part.

6. Define *circle, tangent, angle in a segment.*

If a point be taken within a circle, from which there fall more than two equal straight lines to the circumference, that point is the centre of the circle.

7. If a straight line touch a circle, and from the point of contact a straight line be drawn cutting the circle, the angles which this line makes with the line touching the circle shall be equal to the angles which are in the alternate segments of the circle.

If the bisectors of the adjacent angles A and B of a quadrilateral $ABCD$ meet in E, and the bisectors of the adjacent angles B and C, C and D, D and A meet respectively in F, G, H; show that a circle can be described round the quadrilateral $EFGH$.

8. If from any point without a circle there be drawn two straight lines, one of which cuts the circle, and the other meets it, and if the rectangle contained by the whole line which cuts the circle, and the part of it without the circle, be equal to the square on the line which meets the circle, the line which meets the circle shall touch it.

9. About a given circle, describe a triangle equiangular to a given triangle.

10. Inscribe an equilateral and equiangular hexagon in a given circle.

Show that if $A_1A_2A_3A_4\ldots$ be an equilateral and equiangular figure of 24 sides, the straight lines A_1A_{10} and A_5A_{18} are at right angles.

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11. If two triangles have one angle of the one equal to one angle of the other, and the sides about the equal angles proportionals, the triangles shall be equiangular to one another, and shall have those angles equal which are opposite to the homologous sides.

12. Similar triangles are to one another in the duplicate ratio of their homologous sides.

If C be the centre of a circle, $OPCQ$ a straight line cutting the circle in P and Q, OT a tangent to the circle, and PN another tangent cutting OT in N; show that the triangles OPN and OTC bear the same ratio to one another that OP bears to OQ.

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II. ALGEBRA.

(Up to and including the Binomial Theorem; the theory and use of Logarithms.)

[N.B.—*Great importance will be attached to accuracy.*]

1. Multiply

$$a^3+2a^2b+3ab^2+3b^3 \quad \text{by} \quad a^3-3a^2b+3ab^2.$$

Also show that

$$(x-4y)^3-(y-4x)^3+(2y-3x)^3-(2x-3y)^3=30(x-y)(x+y)^2.$$

2. Divide

$$x^6-(p^4-2pq)x^4-p^2q^2x^2-q^4 \quad \text{by} \quad x^3-p^2x^2+pqx-q^2.$$

3. Find the highest common factor of

$$4x^4-29x^2+25 \quad \text{and} \quad 2x^4+x^3-14x^2-4x+15.$$

Also resolve each of these expressions into the simplest possible factors.

4. Simplify

$$\text{(i.)} \quad \frac{3}{1-x}-\frac{1}{2-x}-\frac{1-2x}{1-x+x^2};$$

$$\text{(ii.)} \quad \frac{a^2}{\sqrt{a^2+b^2}-b}-\frac{b^2}{\sqrt{a^2+b^2}+a}.$$

Also find the value of

$$\frac{1}{x-2a}+\frac{2}{x+b}+\frac{1}{b}, \text{ when } x=\frac{3ab}{b-a}.$$

5. A square of carpet is cut up into strips so as to cover a border $(a+b)$ feet wide round the floor of a room, whose length is $10a$ feet and breadth $10b$ feet. What was the length of the side of the square?

6. Solve the equations

$$\text{(i.)} \quad \frac{2x-3y}{4}=\frac{3x-2y}{5}=7\tfrac{1}{2};$$

$$\text{(ii.)} \quad \frac{\sqrt{3x-2}-\sqrt{x+7}}{\sqrt{3x-2}+\sqrt{x+7}}=\frac{x-3}{2x+1}.$$

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7. Find the sum, product, and difference of the roots of the quadratic equation

$$x^2 - 2bx + c^2 = 0.$$

Write down an equation whose roots are each equal to the sum of the reciprocals of the roots of the given equation.

8. A cask was filled with wine and water mixed together in the proportion of 5 : 3. When 16 gallons of the mixture had been drawn off and the cask filled up with water, the proportion became 3 : 5. How many gallons did the cask hold?

9. The first and last of 46 terms in Arithmetical Progression were -5 and $+25$ respectively. Find the two middle terms and the sum of all the terms.

Also find two numbers whose arithmetic mean exceeds their geometric mean by 2, and whose harmonic mean is one-fifth of the larger number.

10. A purse contains a sovereign, a half-sovereign, a half-crown, a florin, a shilling, a threepenny piece, and a penny. How many different sums of money can be obtained by taking out four of the coins?

If two sovereigns were added to the contents of the purse, how many different sums could then be obtained by taking out four coins?

11. Write down the continued product of the n factors

$$1 + x_1, \quad 1 + x_2, \quad 1 + x_3, \quad 1 + x_4, \ldots\ldots 1 + x_n,$$

and deduce the expansion of $(1 + x)^n$ by the Binomial Theorem.

Also write down the twelfth term and the middle term in the expansion of

$$\left(a - \frac{1}{a}\right)^{16}.$$

12. Explain how the different powers and roots of numbers may be found by means of logarithms.

What is the smallest number of terms which may be taken of the Geometric Progression 4, 6, 9, ... to have a sum exceeding 8000?

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III. PLANE TRIGONOMETRY AND MENSURATION.

(Including the Solution of Triangles.)

[N.B.—*Great importance will be attached to accuracy.*]

1. Compare the magnitudes of the unit of circular measure and the degree.

Find correct to one second, the time between one and half-past one o'clock when the circular measure of the angle between the hands is $\frac{1}{2}$.

2. Express all the trigonometrical functions of an angle in terms of the tangent.

If $$\tan\theta = \frac{x\sin\phi}{1 - x\cos\phi}, \quad \tan\phi = \frac{y\sin\theta}{1 - y\cos\theta},$$

prove that $$\frac{\sin\theta}{\sin\phi} = \frac{x}{y}.$$

3. Find $\sin(A+B)$ in terms of sines and cosines of A and B. Find also $\sin 3A$ and $\cos 2A$ in terms of $\sin A$, and if A is less than $180°$, prove that $\sin 4A$ is less than $4\sin A$.

4. Find the smallest positive values of A and B from the equations

$$\begin{cases} \tan A + \tan B = \cot A + \cot B, \\ A - B = 60°. \end{cases}$$

5. Prove that

$$\sin A - \sin B = 2\sin\frac{A-B}{2}\cos\frac{A+B}{2}.$$

Prove also that

$$\sin 5A - \sin 3A = 2\sin A - 4\sin A\sin^2 2A.$$

6. Find $\tan(A+B+C)$ in terms of the tangents of A, B, and C.

Prove that in any triangle ABC

$$\cot A\cot B + \cot B\cot C + \cot C\cot A = 1.$$

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7. In every triangle the sides are proportional to the sines of the opposite angles.

If the cosines of two of the angles are inversely proportional to the opposite sides, prove that the triangle is either right-angled or isosceles.

8. Explain the notation $\sin^{-1}a$.

If $\tan^{-1}a = \operatorname{cosec}^{-1}a = \cos^{-1}b$, prove that one value of b is

$$\frac{\sqrt{5}-1}{2}.$$

9. Find the area, and radius of the inscribed circle, of a triangle whose sides are 706, 690, and 240 feet.

10. From two stations C and D, on the same side of the line AB, joining the two objects A and B, whose distance apart is known, the angles ACB, BCD, CDB, and ADB are observed, show how CD may be found.

If $ACB = 90°$, $ADB = 75°$, $BCD = 30°$, $CDB = 105°$,

prove that

$$CD = \frac{2AB}{\sqrt{8+2\sqrt{3}}}.$$

11. The height of a conical tent is $7\frac{1}{2}$ feet, and it is to inclose 200 square yards of ground, find how much canvas will be required. ($\pi = \frac{22}{7}$.)

12. The silk covering of an umbrella forms a portion of a sphere of $3\frac{1}{2}$ feet radius, the area of the silk being $14\frac{3}{8}$ square feet. Find the area of the ground sheltered from vertical rain when the stick is held upright. ($\pi = \frac{22}{7}$.)

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IV. STATICS AND DYNAMICS.

[*The acceleration due to gravity may be taken as equal to* 32 *foot-second units.*]

1. If the side of a regular hexagon $ABCDEF$ represents a force of 100 lbs., find, to the nearest pound, the magnitudes of the forces represented by the straight lines AE, AD, FB; and, supposing them to act at a point, show how to draw, by the Polygon of Forces, the line representing their resultant.

2. A uniform rod AB which can turn freely in a vertical plane about a hinge at A, is kept in a horizontal position by a string BC attached to a fixed point C in the vertical plane, the angle ABC being obtuse. Show in a diagram the forces acting on the rod, and prove that two of them are equal.

3. State the conditions of equilibrium of a system of forces acting in one plane on a particle (i.) when the particle is otherwise free, and (ii.) when it is constrained to move on a smooth straight rod in the plane of the forces.

A ring C rests upon a fixed smooth horizontal rod AB, whose length is 13 feet. To the ring are attached two strings, one of which is 7 feet long and has its other extremity fixed at A, the other passes over a smooth hook 8 feet below B and supports a weight of 20 lbs. Find the tension of the string AC.

4. Define the *centre of gravity* of a body; and show how to find the position of the centre of gravity of a body formed of two parts whose weights and centres of gravity are given.

A line BOC is bisected at right angles by a line AOD, the lengths of AO, OD being 5 and 7 inches respectively; find the distance of the centre of gravity of the figure $ABDC$ from O.

5. Point out the use of a single smooth pulley (i.) when fixed, (ii.) when moveable, and in the latter case investigate the relation between the power and the weight when the angle between the portions of the string diverging from the pulley is α. Within what limits must α lie in order that there may be mechanical advantage?

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6. Two weightless levers AOB, COD, whose lengths are 8 and 9 inches respectively, are jointed together at O four inches from B and D. If A, C be connected by a string 3 inches long, and B, D be pulled apart by forces P, P, in the straight line BD, find the tension of the string.

Draw a diagram showing the forces acting on the lever AOB.

7. Enunciate the First and Second Laws of Motion, and give examples of force producing change of motion as regards (i.) magnitude only, (ii.) direction only, and (iii.) both magnitude and direction.

8. Compare the velocities of two particles, one of which describes $4\frac{1}{2}$ miles in 35 minutes, and the other 77 feet in 2 seconds.

9. A particle moves from rest under the action of a uniform force; express the velocity acquired, the acceleration, and the force, in terms of s the space described, t the time, and m the mass of the particle.

A smooth tube ACB, consisting of two straight portions AC, CB, with a bend at its lowest point C, is fixed in a vertical plane, and a particle acted on by gravity starts from rest from a point P in the arm AC, and, passing the bend C without change of velocity, rises in the arm CB to a point Q. Show that PQ is horizontal, and that the spaces CP, CQ are proportional to the times in which they are described.

10. If a force equal to the weight of 21 lbs. act upon a body, and generate in it in one minute a velocity of 3 feet per second, find the mass of the body.

11. A particle is projected with a velocity of 80 feet per second in a direction making an angle of 30° with the horizon; find the greatest height attained, and the range on the horizontal plane through the point of projection.

Find also the velocity of the particle and its distance from the point of projection at the end of two seconds.

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FURTHER EXAMINATION.

V. PURE MATHEMATICS.

[*Full marks may be obtained for about* three-fifths *of this paper. No Candidate must attempt more than* ten *questions. Great importance will be attached to accuracy.*]

1. Prove that the sum of the squares of any three real quantities, which are not all equal, is greater than the sum of the products of every two of the quantities.

2. Expand

$$\frac{(1-x)^2}{1+x+x^2}$$

in a series of ascending powers of x.

If a_r be the coefficient of x^r in this expansion, prove that

$$a_{3n} = 0, \quad a_{3n+2} - a_{3n+1} = 6.$$

3. Find two numbers whose product exceeds the square of the smaller by the double of the greater. (Only one solution is required.)

4. In a certain triangle ABC the base AB and the perpendicular altitude are each equal to the radius of the circumscribing circle. Prove that

$$ab = 2c^2 \quad \text{and} \quad \cot A + \cot B = 1.$$

5. A vertical column stands on an inclined plane. The elevation of the top of the column above the horizon is found to be 45° at points on this plane respectively a ft. and b ft. from the foot of the column along the line of greatest slope through the foot. Show that the inclination of the plane to the horizon is

$$\tan^{-1}\left(\frac{a-b}{a+b}\right),$$

and that the square of the height of the column is the harmonic mean between a^2 and b^2.

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6. Show that θ, the circular measure of an angle not exceeding half a right angle, is equal to the sum of either of the infinite series

$$\tan\theta - \tfrac{1}{3}\tan^3\theta + \tfrac{1}{5}\tan^5\theta - \tfrac{1}{7}\tan^7\theta + \dots,$$

$$\sin 2\theta - \tfrac{1}{2}\sin 4\theta + \tfrac{1}{3}\sin 6\theta - \tfrac{1}{4}\sin 8\theta + \dots.$$

7. The tangent at any point P of a conic section, whose focus is S, meets the directrix in D. Show that the angle PSD is a right angle.

If in a parabola the abscissa of P be one-twelfth of the latus rectum, PD will be bisected by the axis.

8. If SY, $S'Y'$ be the perpendiculars from the foci of an ellipse on the tangent at a point P, show that $SY \cdot S'Y' = BC^2$.

Show also that, if the circle on YY' as diameter cut the normal at P in R, and PN be the ordinate at P, $PN : PR :: BC : SC$.

9. Prove that the difference of the squares of any two conjugate diameters of a hyperbola is constant.

If the base and one of the equal sides of an isosceles triangle be the asymptotes of a hyperbola which touches the other side, prove that the difference of the squares of any two conjugate diameters is double of the square of the base of the triangle.

10. Express the area of a triangle in terms of the rectangular co-ordinates of its angular points.

Hence find the area of the triangle formed by joining the points, whose coordinates are $(a, c+a)$, (a, c), $(-a, c-a)$, and verify your result geometrically.

11. AOA', BOB' are two perpendicular diameters of a circle, and M, N are the middle points of OA, OB respectively. Taking these diameters as axes of coordinates, write down (1) the equations of the straight lines AN, $B'M$; (2) the equations of the radii drawn to the two points where AN, $B'M$ produced cut the circumference of the circle.

What is the angle between the two radii?

12. Prove that the polar equation of a parabola, with the focus as pole, may be written in the form

$$r\cos^2\frac{\theta}{2} = a.$$

Prove also that the equation of the *polar* of a point, whose coordinates are (r', θ'), is

$$rr'\sin\theta\sin\theta' + 2a(r\cos\theta + r'\cos\theta') = 4a^2.$$

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13. Find the equation of the straight line joining two points on an ellipse whose eccentric angles are given.

If tangents are drawn to the ellipse at two points, whose eccentric angles differ by 120°, show that their point of intersection will always lie on a similar and similarly situated ellipse, whose axes are twice as long as those of the given ellipse.

14. Find the equation of a hyperbola, of eccentricity $\frac{5}{4}$, which has one focus at the point $(a, 0)$, and the equation of the corresponding directrix $4x - 3y = a$.

Find also the coordinates of the centre, and the equation of the other directrix.

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VI. MECHANICS.

[*Full marks may be obtained for about* two-thirds *of this paper. Great importance is attached to accuracy.* N.B.—*g may be taken* = 32.]

1. If OA and OB represent two forces in direction and magnitude, find the line OR representing their resultant.

If OC and OD be two equal lines cut off from OA and OB respectively, and if OR meet CD in G, find the ratio of CG to GD.

2. A rod AB, whose centre of gravity is at G, is supported at an angle of 42° to the vertical, with one end A in contact with the smooth vertical wall AD, by a string attached to the point C of the rod and also to a point in the wall.

$$AB = 3'', \quad AC = 1'', \quad AG = 1\tfrac{5}{8}''.$$

Find graphically the length of the string and its tension in terms of the weight of the rod, which is 55 ozs.

3. The horizontal rod BEC and the rod DE being without weight, and the latter perfectly rigid, find graphically the actions at B and E when the weight 150 lbs. is suspended from C. BD is vertical and

$$BC = 1{\cdot}8, \quad BE = {\cdot}9, \quad DE = 1{\cdot}5, \quad BD = 1{\cdot}2.$$

4. Find the relation between the power and the weight in the smooth wedge whose transverse section is an isosceles triangle. If the pressure between the wedge and the separated points in contact with it be proportional to the distance between these points, prove that the work done is proportional to the square of the distance of penetration.

5. The uniform square $ABCD$ rests vertically with the side BC upon a horizontal plane, coefficient of friction $\frac{1}{2}$, and has a rope attached at D and passing over a small smooth pulley at the point E in BA produced till EA is equal to AB. If the rope be pulled, find whether the initial motion of the square will be tilting or sliding.

6. Apply the laws of motion to determine the path of a projectile.

From a given point in a railway carriage, moving with uniform velocity in a straight line, bullets are fired continuously with a constant velocity at right angles to the rails, and with a constant inclination to the horizon. Find the locus at any instant of all the bullets which have not reached the ground.

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7. Find the acceleration down a smooth inclined straight line.

Two particles slide down two straight lines, in the same vertical plane, at right angles to one another, starting simultaneously from their point of intersection; prove that their distance apart, at any time, will be equal to the distance either would have descended vertically in that time.

8. What is the experimental principle assumed in determining the velocities of two given elastic balls after direct impact? If the elasticity is perfect, prove that the total kinetic energy is unaltered by impact.

Prove also the converse of this proposition.

9. Two equal and perfectly elastic spherical beads, each of radius r, strung upon an inextensible string, are placed on a smooth table and are drawn apart to the greatest possible distance a between their centres. One of them is then projected directly towards the other with a given velocity u. Determine their distance apart, and their velocities, at any time t from projection.

Write down the equation to the path of their centre of gravity, supposing the table removed at the instant of projection.

10. Two given weights are connected by a string passing over a smooth pulley. Determine their acceleration and prove that the resultant pressure between the string and pulley is less than it would have been if half the sum of the weights had been suspended at each end of the string.

11. Define the terms Work, and Energy (potential and kinetic), stating the units commonly used in each case.

A shot of 6 lbs. weight leaves the muzzle of a gun of 6 cwt. in a horizontal direction with the velocity of 1000 feet per second. Find the potential energy of the charge in the ordinary units, assuming that the gun is perfectly rigid, its bore smooth, and the carriage free to move on a horizontal plane.

How would the conclusion be affected in the absence of any one of these assumptions.

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MATHEMATICAL EXAMINATION PAPERS

FOR ADMISSION INTO

Royal Military Academy, Woolwich,

NOVEMBER, 1892.

OBLIGATORY EXAMINATION.

I. EUCLID (BOOKS I.—IV. AND VI.).

[*Ordinary abbreviations may be employed, but the method of proof must be geometrical. Proofs other than Euclid's must not violate Euclid's sequence of propositions. Great importance will be attached to accuracy.*]

1. Make a triangle of which the sides shall be equal to three given straight lines; and show that if ABC be a triangle with the vertical angle at A greater than either of the base angles, it is not possible to form a second triangle with sides equal to those of ABC and base equal to twice BC.

2. Give Euclid's *Axiom on Parallels*, and those on the addition and subtraction of equals to and from equals and unequals.

Prove that if a straight line fall on two parallel straight lines, it makes the alternate angles equal to one another; and the exterior angle equal to the interior and opposite angle on the same side; and also the two interior angles on the same side together equal to two right angles.

3. ABC being an isosceles triangle, and D any point in the base BC; show that the perpendiculars to BC through the middle points of BD and DC divide AB and AC at H and K respectively, so that $BH = AK$ and $AH = CK$.

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4. If a straight line be bisected, and produced to any point, the rectangle contained by the whole line thus produced, and the part of it produced, together with the square on half the line bisected, is equal to the square on the straight line which is made up of the half and the part produced.

Prove this; and give its equivalent in algebraical symbols.

5. Prove that in every triangle, the square on the side subtending an acute angle, is less than the squares on the sides containing that angle, by twice the rectangle contained by either of these sides, and the straight line intercepted between the perpendicular let fall on it from the opposite angle, and the acute angle.

If in a quadrilateral $ABCD$, the square on AB is greater than the squares on the three other sides BC, CD, DA by twice the rectangle CD, DA, show that the angle ACB is obtuse.

6. Prove that the straight line drawn at right angles to the diameter of a circle, from the extremity of it, falls without the circle; and that no straight line can be drawn from the extremity, between that straight line and the circumference, so as not to cut the circle.

Give the corollary to this proposition.

7. Prove that the angle at the centre of a circle is double of the angle at the circumference on the same base, that is, on the same arc.

If the diagonals of a quadrilateral inscribed in a circle cut at right angles, show that the angles which a pair of opposite sides of the quadrilateral subtend at the centre of the circle are supplementary.

8. If from any point without a circle two straight lines be drawn, one of which cuts the circle, and the other touches it; prove that the rectangle contained by the whole line which cuts the circle, and the part of it without the circle, is equal to the square on the line which touches it.

9. Inscribe a circle in a given triangle; and indicate the construction when the circle is required to touch one side externally.

10. Inscribe a regular pentagon in a given circle.

If a regular pentagon and a regular hexagon be on the same base and on the same side of it, prove that the pentagon is wholly within the hexagon.

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11. In a right-angled triangle, if a perpendicular be drawn from the right angle to the base, prove that the triangles on each side of it are similar to the whole triangle, and to one another.

If AB, BC be two sides of a regular figure, L and M their respective middle points, and O the centre of the inscribed circle, show that the triangle BLM has to the triangle OLM the duplicate ratio of that which a side of the figure has to the diameter of the circle.

12. Prove that in equal circles, angles, whether at the centres or circumferences, have the same ratio which the circumferences on which they stand have to one another.

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II. ALGEBRA.

(Up to and including the Binomial Theorem, the theory and use of Logarithms.)

[N.B.—*Great importance will be attached to accuracy.*]

1. Multiply

$$4x^2+2(3a-2b)x+9a^2 \quad \text{by} \quad 2x-3a,$$

and divide

$$16a^4-8a^2bc+b^2c^2 \quad \text{by} \quad 2a-\sqrt{bc}.$$

2. Extract the square root of

$$9a^6-12a^5b+\frac{10a^3b^3}{3}-\frac{4ab^5}{27}+\frac{b^6}{81}.$$

3. Define the meaning of the symbol $\frac{a}{b}$, and from your definition show that $\frac{a}{b}=\frac{na}{nb}$, where a, b, n are positive integers.

Arrange in order of magnitude, a, b, n being positive integers,

$$\frac{na}{(n+1)b}, \quad \frac{(n+1)a}{nb}, \quad \frac{na}{(n-1)b}, \quad \frac{(n-1)a}{nb},$$

and justify your arrangement.

4. Simplify

(i.) $\sqrt[m]{a^{m^2}\div(a^m)^2}$,

(ii.) $$\left\{\frac{1}{2x^2-3ax-2a^2}+\frac{1}{2x^2-5ax+2a^2}\right\} \div \left\{\frac{1}{2x^2+3ax-2a^2}-\frac{1}{2x^2+5ax+2a^2}\right\},$$

(iii.) $$\frac{a^3-b^3}{a-b+\dfrac{2(a+b)}{\dfrac{a+b}{b}-\dfrac{a}{a-b}}}.$$

5. Find the value of

$$\left(\frac{x+a}{x-a}\right)^2-\frac{x}{3a} \quad \text{when} \quad x=a(1+2\sqrt{3}),$$

and reduce

$$\sqrt{6-\sqrt{17-12\sqrt{2}}}$$

to the simplest possible surd form.

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6. Solve the equations

(i.) $\dfrac{x}{x+2}-\dfrac{3x-10}{3x-9}=\dfrac{5}{3-x}$,

(ii.) $2ax-by=a^2, \quad bxy=a^3$,

(iii.) $\sqrt{5x-14}+\sqrt{3x-13}=2\sqrt{x}+\sqrt{2x+1}$.

7. Prove that a quadratic equation cannot be satisfied by more than two values of the unknown quantity.

In a certain quadratic the coefficients of x^2 and x are 1 and 2 respectively, and the addition of 8 to each of the roots changes the sign but not the magnitude of the third term. Find the original quadratic, and the coefficient of x in the transformed equation.

8. Insert five arithmetic means between $a-2b$ and $3a+b$.

The last term of an arithmetic progression is ten times the first, and the last but one is equal to the sum of the 4th and 5th. Find the number of the terms, and show that the common difference is equal to the first term.

9. Find the number of arrangements that can be made of the letters of the word *infinite*, (1) when they are taken all together, (2) when they are taken four together so that each arrangement has two vowels and two consonants.

10. Write down the first five terms in the expansion of $(1-4x)^{\frac{1}{2}}$ by the Binomial Theorem, and show that the coefficient of x^n may be written in the form

$$-\frac{2\,.\,\underline{|2n-2}}{\underline{|n}\,.\,\underline{|n-1}}.$$

11. Prove that

$$\log 20+7\log\left(\frac{15}{16}\right)+5\log\left(\frac{24}{25}\right)+3\log\left(\frac{80}{81}\right)=1,$$

and, by means of the logarithm tables supplied, find the fifth root of

$$\frac{66901\times 337}{7824}$$

correct to five places of decimals.

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III. PLANE TRIGONOMETRY AND MENSURATION.

(Including the Solution of Triangles.)

[N.B.—*Great importance will be attached to accuracy.*]

1. Given that $\sin A = \frac{12}{37}$; find the other trigonometrical ratios of A.

2. Trace the changes in magnitude and sign of

$$\text{(i.) } \cot\theta, \quad \text{and (ii.) } \cos\theta - \sqrt{3}\sin\theta,$$

as θ varies from 0 to 2π.

3. Prove that

$$\text{(i.)} \quad \sec^2 A + \operatorname{cosec}^2 A = \sec^2 A \,.\, \operatorname{cosec}^2 A\,;$$

$$\text{(ii.)} \quad \frac{\sin^3 A + \cos^3 A}{\sin A + \cos A} + \frac{\sin^3 A - \cos^3 A}{\sin A - \cos A} = 2\,;$$

$$\text{(iii.)} \quad \tan\left(\frac{\pi}{4} + A\right) - \tan\left(\frac{\pi}{4} - A\right) = 2\tan 2A.$$

4. Obtain a formula giving the value of $\sin\frac{A}{2}$ in terms of $\sin A$. Show how to get rid of the ambiguities of sign when A lies between $270°$ and $450°$.

5. Prove that

$$\text{(i.)} \quad \cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2},$$

$$\text{(ii.)} \quad \cos B - \cos A = 2\sin\frac{A+B}{2}\sin\frac{A-B}{2}.$$

Prove also that

$$\text{(i.)} \quad \sin A\sin(B-C) + \sin B\sin(C-A) + \sin C\sin(A-B) = 0,$$

$$\text{(ii.)} \quad \sin(B-C)\sin(B+C-2A) + \sin(C-A)\sin(C+A-2B) + \sin(A-B)\sin(A+B-2C) = 0.$$

6. Find an expression for $\sin 72°$, and calculate its value correctly to three places of decimals.

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7. Prove that

$$\tan^{-1}\frac{2mn}{m^2-n^2}+\tan^{-1}\frac{2pq}{p^2-q^2}=\tan^{-1}\frac{2MN}{M^2-N^2},$$

where $$M=mp-nq, \quad N=np+mq.$$

8. Explain how to solve a triangle, having given two sides and the included angle.

Given that $b=23$, $c=32$, $A=63°$; find a as nearly as you can, by aid of the logarithm tables supplied.

9. Find an expression for $\tan\frac{A}{2}$ in terms of the sides of the triangle ABC.

Prove that, if Δ is the area of the triangle ABC,

$$\frac{\tan\frac{A}{2}}{(a-b)(a-c)}+\frac{\tan\frac{B}{2}}{(b-c)(b-a)}+\frac{\tan\frac{C}{2}}{(c-a)(c-b)}=\frac{1}{\Delta}.$$

10. The angle of elevation of the top of a tower at a certain spot is 55°, and at a spot, in the same horizontal plane, 25 yards further from the tower the angle of elevation is 47°. Find the height of the tower.

11. Find the area of the surface (including the ends) of a hexagonal prism, whose height is 8 ft., the base being a regular hexagon with a side of length 3 ft.

12. The radii of the internal and external surfaces of a hollow spherical shell of metal are 3 ft. and 5 ft. respectively. If it be melted down and the material formed into a cube, find an approximate value for the length of an edge of the cube.

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IV. STATICS AND DYNAMICS.

[*The acceleration due to gravity may be taken as equal to* 32 *foot-second units.*]

1. Explain how the resultant of two known forces acting on a particle may be determined by a geometrical construction.

If D be the middle point of the base BC of a triangle ABC, and the resultant of the forces represented by BA, BD be equal to the resultant of those represented by CA, CD, show that the triangle ABC is isosceles.

2. A, B, C are three points in a straight line ABC. Draw a diagram representing the directions of three parallel forces P, Q, R in equilibrium acting at these points respectively. State also the necessary and sufficient conditions of equilibrium.

A uniform rod, 2 feet long and weighing 3 lbs., lies on a horizontal plane; find the least force which, applied 5 inches from one end, will raise that end above the plane.

3. Define the moment of a force about a point, and show how it can be geometrically represented.

The side BC of a triangle ABC is bisected in E and produced to F. Show that the sum of the moments about F of the forces represented by AB, AC is equal to twice the moment about F of the force represented by AE.

4. Show that the centre of gravity of a triangular lamina coincides with that of three equal weights placed at its angular points.

A lamina in the form of a right-angled triangle ABC is suspended from the right angle C. If $CA = 2$ feet, and $CB = 3$ feet, find the weight which must be suspended from A in order that AB may be horizontal in the position of equilibrium, the weight of the lamina being 12 lbs.

5. A smooth rod BC is passed through a small ring and placed upon a horizontal plane, with its ends attached to a fixed point A in the plane by two strings AB, AC, which are tight. A horizontal force being applied to the ring, find its direction, and also the position of the ring on the rod, in order that equilibrium may not be disturbed, the lengths of BC, CA, AB being 25, 20, and 15 inches respectively.

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6. Define a machine, and point out its use as regards work done by it.

Find the greatest vertical height through which a force of 150 lbs. can raise a weight of 4 cwt. by drawing it up a smooth, sloping plank 20 feet in length.

7. Prove that, if an insect crawl along the minute hand of a clock with a velocity equal to that of the extremity of the hand, it will pass from one end to the other in 9 minutes 33 seconds nearly.

8. Explain, with examples, what is meant by a particle having two or more velocities in different directions at the same instant.

A particle has two velocities, $3u$ in a direction from A to B, and $8u$ in a direction from C to A, ABC being an equilateral triangle. Find the magnitude of its resultant velocity.

9. State the expression for the space described in time t by a particle projected with velocity u, and subject to an acceleration a in the direction of projection. If v be the final velocity, find the space in terms of v, t, and a.

If $u = 20$ and $a = 12$, the units of time and space being one second and one foot, find the space described in the 4th second.

10. What is meant by the statement that g, the acceleration due to gravity, is equal to 32 nearly?

Show how you would roughly obtain the value of g by observing the motion of two weights P, Q, connected by a string passing round a fixed smooth pulley.

Describe fully a single observation which would give the value of g; P, Q being 47 and 49 ounces respectively.

11. A particle is projected in any direction not vertical; explain clearly why it does not proceed to describe a straight line.

The engine driver of an express train throws a ball vertically upwards; show in a diagram the actual path of the ball.

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FURTHER EXAMINATION.

V. PURE MATHEMATICS.

[*Full marks may be obtained for about* three-fifths *of this paper. No Candidate must attempt more than* ten *questions. Great importance will be attached to accuracy.*]

1. A certain council consists of a chairman, two vice-chairmen, and twelve other members. How many different committees of six members can be chosen, including always the chairman and *at least* one of the vice-chairmen?

Show that 8190 is the total number of different committees which can be formed consisting of the chairman, one (and only one) of the vice-chairmen, and at least one of the other twelve members.

2. Express as a continued fraction the quotient obtained by dividing

$$\frac{1}{a^2 - \frac{1}{2a^2 -}\ \frac{1}{2a^2 -}\ \frac{1}{2a^2 -}\cdots}\cdots \quad \text{by} \quad \frac{1}{a - \frac{1}{2a -}\ \frac{1}{2a -}\ \frac{1}{2a -}\cdots}.$$

3. There are six balls in a bag, each of which is known to be either white or black. The first five balls drawn are three white and two black. What is the chance that the other ball is white?

If the five balls were replaced, and a second drawing gave also three white and two black. What would the chance then be?

4. A hill in the shape of a right cone stands on a horizontal plane. At a certain point in the plane the circular base of the cone subtends a right angle, and the elevation of the summit is half a right angle. Show that the slant side of the hill, as seen against the sky, subtends 60° at the same point.

5. The lengths of the sides of a right-angled triangle are in arithmetical progression. Find the ratios of the sides, and show that the diameters of the escribed circles are in harmonical progression.

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6. Assuming the truth of De Moivre's theorem, prove that

$$\sin\theta = \theta - \frac{\theta^3}{\lfloor 3} + \frac{\theta^5}{\lfloor 5} - \text{etc.}$$

Also prove that

$$\theta = x\sin\theta + \frac{x^2\sin 2\theta}{2} + \frac{x^3\sin 3\theta}{3} + \text{etc.},$$

where $2x = \sec\theta$.

Are these formulæ true for all values of θ?

7. Show how to describe a conic section, whose focus, directrix, and eccentricity are given.

If a series of conics pass through two fixed points, P, Q, and if a directrix of each pass through a fixed point in PQ produced, prove that the corresponding foci of the conics all lie on a fixed circle.

8. If the ordinate, tangent, and normal at any point of an ellipse meet the major axis in N, T, G, respectively, prove that

(i.) $NG : CN :: BC^2 : AC^2$.

(ii.) SC is a mean proportional between CG and CT.

9. Prove that the latus rectum of an hyperbola is a third proportional to the transverse and conjugate axes.

If SY, the perpendicular from the focus upon an asymptote, be produced to meet the conjugate axis in W, prove that YW is a third proportional to the conjugate and transverse semi-axes.

10. Show how to express the position of a point on a plane by means of polar coordinates.

Indicate in a figure the four points whose polar coordinates are

$$\left(a, \frac{\pi}{3}\right), \quad \left(2a, -\frac{\pi}{2}\right), \quad \left(-a, \frac{\pi}{6}\right), \quad \text{and} \quad \left(-2a, -\frac{2\pi}{3}\right),$$

respectively, and also the locus of the equation $r(4 - 3\sin^2\theta) = 8a\cos\theta$.

11. Find the equations of the two straight lines drawn through the point $(0, a)$, on which the perpendiculars let fall from the point $(2a, 2a)$ are each of length a.

Show that the equation of the straight line joining the feet of these perpendiculars is

$$y + 2x = 5a.$$

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12. Define the terms *pole* and *polar* with respect to a conic section, and find the equation of the polar of any point with respect to a parabola.

If a perpendicular be let fall from the pole upon the polar, prove that the distance of the foot of this perpendicular from the focus is equal to the distance of the pole from the directrix.

13. Find the equation of the tangent at any point of the ellipse

$$a^2y^2 + b^2x^2 = a^2b^2.$$

If the tangent at P have intercepts f, g upon the coordinate axes, prove that

$$\frac{a^2}{f^2} + \frac{b^2}{g^2} = 1.$$

14. Investigate the condition that $y = mx$, $y = m'x$ may be conjugate diameters of the hyperbola $b^2x^2 - a^2y^2 = a^2b^2$.

Find the equation of that diameter of the hyperbola $2x^2 + xy - y^2 = 9a^2$, which is conjugate to the diameter coinciding with the axis of x, and prove that the difference of the squares of these two conjugate diameters is equal to $16a^2$.

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VI. MECHANICS.

[*Full marks may be obtained for about* two-thirds *of this paper. Great importance is attached to accuracy.* N.B.—*g may be taken* = 32.]

1. Two equal weightless rods, AB and AC, are hinged together at A at an angle of 74°, and placed in a vertical plane, with B and C on a smooth horizontal plane. If B and C be connected by a string, and a weight of 30 lbs. be suspended at D, where $AD : DC = 5 : 4$, find graphically the tension of the string.

2. Four equal weightless rods are hinged together to form the rhombus $ABCD$, and the hinges A and C are connected by a string. If the rhombus be suspended from A, and equal weights of 1 cwt. each be suspended from B and D, find graphically the tension of the string.

3. Four equal weightless rods are hinged together to form the rhombus $ABCD$, and the hinges at B and D are joined by the weightless rod BD. The angle $BAD = 64°$. If the rhombus be suspended from A, and a weight of 1 cwt. be suspended from C, find the thrust in BD.

4. Prove that the centre of gravity of a triangle coincides with that of three equal particles at the angles.

The uniform plane quadrilateral $ABCD$ right-angled at A and obtuse-angled at B is bisected by the diagonal AC. If it be placed with its plane vertical and the side AB on a horizontal plane, prove that the condition of equilibrium is $2BM < 3AB$, where M is the foot of the vertical from C on AB produced.

5. Explain the construction and action of the screw, and find the relation of the power to the weight when there is no friction.

6. A uniform beam, weight W, laid on a horizontal plane, can be just moved by pushing it with a horizontal force $\frac{1}{3} W\sqrt{3}$. Prove that the least force which can move it is equal to $\frac{W}{2}$, and find the direction of this force.

If the beam be pulled slowly by a rope attached at one end A, prove that A will not rise from the ground unless the inclination of the rope to the horizontal be 60° at least.

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7. A body is projected from a given point O, with the velocity v, at the angle α to the horizon; find its distance from O after the time t.

If any number of such particles be projected from O with the same velocity in different directions, prove that at any subsequent instant they will all lie on the surface of a sphere.

8. Find the horizontal and vertical accelerations of a body sliding down a smooth inclined plane.

If two bodies start at the same instant sliding down two lines in the same vertical plane sloping towards the same direction, at angles α and β to the horizon, prove that each as seen from the other will always appear to be moving parallel to a line inclined to the horizon at the angle $\alpha+\beta$.

9. Find the velocities of two elastic spheres of given masses and elasticities after direct impact.

If two perfectly elastic smooth spheres, A and B, impinge upon each other directly or obliquely, prove that B's velocity relative to A, after impact, will make the same angle with the line of centres as the relative velocity of A to B did before impact.

10. Find the acceleration of a particle describing a circle, radius a, with velocity v.

A circus horse gallops round a circle of 30 feet radius, at a speed of 15 miles an hour, prove that the least value of the coefficient of friction between feet and ground, that the horse may not slip, neglecting the distance between the feet and centre of gravity, is $\frac{1}{2}$ very nearly.

11. A train weighs M tons, and the resistance of friction is p lbs. per ton. If the engine can exert a pull of P lbs., and the break a resistance of R lbs., find the distances passed over in attaining a speed of v miles per hour from rest, and in slowing down from that speed to rest respectively moving on the level.

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MATHEMATICAL EXAMINATION PAPERS

FOR ADMISSION INTO

Royal Military Academy, Woolwich,

JUNE, 1893.

OBLIGATORY EXAMINATION.

I. EUCLID (BOOKS I.—IV. AND VI.).

[*Ordinary abbreviations may be employed; but the method of proof must be geometrical. Proofs other than Euclid's must not violate Euclid's sequence of propositions. Great importance will be attached to accuracy.*]

1. Define *right angle, trapezium, quadrilateral.*

If two triangles have two sides of the one equal to two sides of the other, each to each, but the angle contained by the two sides of one of them greater than the angle contained by the two sides equal to them of the other, the base of that which has the greater angle shall be greater than the base of the other.

$ABCD$, $EFGH$ are quadrilaterals in which the sides AB, BC, EF, FG are all equal to one another; and the angles at C, D, G, H are right angles. Show that if the angle at F is greater than that at B, then is EH greater than AD.

2. What is the magnitude of the angle of a regular figure of n sides?

Three regular figures, of n_1, n_2, and n_3 sides, have one vertex in common; and just fill the whole space at that vertex; show that

$$\frac{1}{n_1}+\frac{1}{n_2}+\frac{1}{n_3}=\frac{1}{2}.$$

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3. Describe a square on a given straight line. What does the demonstration indicate concerning a parallelogram which has a right angle?

4. If a straight line be divided into any two parts, the square on the whole line is equal to the squares on the two parts, together with twice the rectangle contained by the two parts.

What is the equivalent of this in algebraical symbols?

5. If in the triangle ABC, which has the obtuse angle ACB, AD be drawn perpendicular to BC produced, the square on AB is greater than the squares on AC and CB by twice the rectangle BC, CD. Prove this, and give the value for the square on AC in terms of the sum of the squares on AB and BC and a rectangle under segments of the base.

In a quadrilateral $ABCD$, AC is equal to CD, AD is equal to BC, and the angle ACB is supplementary to the angle ADC; show that the square on AB is equal to the sum of the squares on BC, CD, DA.

6. If two circles touch one another externally, the straight line which joins their centres shall pass through the point of contact.

7. The opposite angles of any quadrilateral figure inscribed in a circle are together equal to two right angles.

ABC is an isosceles triangle in which AB is equal to AC; and AD is drawn to meet the base BC in D; show that the centre of the circle described round ABD is at the same distance from AB that the centre of the circle round ADC is from AC.

8. Define *tangent, similar segments of circles.*

If a straight line touch a circle, and from the point of contact a straight line be drawn cutting the circle, the angles which this line makes with the line touching the circle shall be equal to the angles which are in the alternate segments of the circle.

9. When are rectilineal figures said to be *inscribed in* and *described about* a circle?

About a given circle describe a triangle equiangular to a given triangle.

10. Inscribe an equilateral and equiangular hexagon in a given circle.

Show that the regular hexagon inscribed in a circle is three-fourths of the regular hexagon described about the same circle.

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11. If the sides of two triangles, about each of their angles, be proportionals, the triangles shall be equiangular to one another and shall have those angles equal which are opposite to the homologous sides.

12. Similar triangles are to one another in the duplicate ratio of their homologous sides.

ABC is a triangle with an obtuse angle at A; and AM, AN are drawn to meet the base BC in M and N respectively, so that the angles AMB, ANC are each equal to BAC; show that BM has to NC the duplicate ratio of that which AB has to AC.

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II. ALGEBRA.

(Up to and including the Binomial Theorem, the theory and use of Logarithms.)

[N.B.—*Great importance will be attached to accuracy.*]

1. Resolve into real elementary factors

$$6x^2-23xy+20y^2,\quad x^4-7x^2y^2+y^4,\quad x^6-1.$$

2. Divide $(x+y)^5-x^5-y^5$ by x^2+xy+y^2.

3. Find the Highest Common Factor of the expressions

$$x^4-3x^3+10x^2-11x+15 \text{ and } 3x^4-7x^3+15x^2-x-10.$$

4. Simplify

(i.) $$\frac{9}{x^2-x-20}-\frac{7}{x^2+x-12}-\frac{2}{x^2-8x+15},$$

(ii.) $$\frac{\dfrac{1}{x-a}-\dfrac{1}{x+a}-\dfrac{2a}{x^2+a^2}}{\dfrac{1}{x^3-a^3}-\dfrac{1}{x^3+a^3}}\left(\frac{1}{x^2+ax+a^2}+\frac{1}{x^2-ax+a^2}\right),$$

(iii.) $$\frac{\sqrt{3}}{\sqrt{3}+\sqrt{5}+\sqrt{7}}-\frac{\sqrt{3}}{\sqrt{5}+\sqrt{7}-\sqrt{3}}+\frac{\sqrt{3}}{\sqrt{7}+\sqrt{3}-\sqrt{5}}+\frac{\sqrt{3}}{\sqrt{3}+\sqrt{5}-\sqrt{7}}.$$

5. Obtain the square root of each of the expressions:

(i.) $9x^4-42x^3+37x^2+28x+4$,

(ii.) $9-\sqrt{77}$.

6. Prove that the form of the expression

$$\frac{(x^4-4x^3+8x-4)^2}{(x^2-2x+2)^4}$$

is unaltered by the substitution of $\dfrac{2}{2-x}$ for x.

7. Solve the equations

(i.) $$\frac{3x+5}{13}-\frac{5x-2}{11}=\frac{4x-3}{5}-\frac{5x+1}{6},$$

(ii.) $$\sqrt{x+2}+\sqrt{x-3}=\sqrt{3x+4},$$

(iii.) $$x^2+xy+y^2=19,\quad x^4+x^2y^2+y^4=133.$$

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8. If α and β be the roots of the quadratic equation

$$ax^2+bx+c=0,$$

prove that

$$\alpha+\beta=-\frac{b}{a}, \text{ and } \alpha\beta=\frac{c}{a};$$

and also construct the quadratic whose roots are

$$\frac{1-\alpha}{1+\alpha} \text{ and } \frac{1-\beta}{1+\beta}.$$

9. The sum of five numbers in arithmetical progression is 10, and the sum of their squares is 60; find the numbers.

10. Prove that if

$$ma^3+nc^3 : pa^3+qc^3 :: mb^3+nd^3 : pb^3+qd^3,$$

then either

$$a : b :: c : d \quad \text{or} \quad m : n :: p : q.$$

11. Write down the first negative term in the expansion of $(1+x)^{\frac{27}{8}}$ with the aid of the Binomial Theorem, taking x to be a positive quantity.

12. Explain how to deduce the value of the logarithm of a number to any given base, from the logarithm of that number to base 10.

Find, with the aid of the logarithmic tables supplied, an approximate value for the incommensurable root of the equation

$$3^{2x}-5\times 3^x+6=0.$$

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III. PLANE TRIGONOMETRY AND MENSURATION.

(Including the Solution of Triangles.)

[N.B.—*Great importance will be attached to accuracy.*]

1. Define the circular measure of an angle, and, assuming that $\pi = \frac{22}{7}$, find, to three places of decimals, the number of degrees in the unit of circular measure.

Also find, to three places of decimals, the number of degrees in the angle whose circular measure is ·63.

2. Write down the expressions for $\sin(A+B)$ and $\cos(A+B)$ in terms of the sines and cosines of A and B, and prove the formula

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}.$$

Hence prove that

$$\tan 15^\circ = 2 - \sqrt{3} \quad \text{and} \quad \tan 75^\circ = 2 + \sqrt{3}.$$

3. If $\sin A = \frac{9}{17}$, find the values of $\sin 2A$, $\cos 2A$, and $\tan 2A$.

4. Prove the formulæ

(i.) $\operatorname{cosec} 2A = \cot A - \cot 2A.$

(ii.) $\sin^2\left(\frac{\pi}{8} + \frac{\theta}{2}\right) - \sin^2\left(\frac{\pi}{8} - \frac{\theta}{2}\right) = \frac{1}{\sqrt{2}} \sin\theta.$

(iii.) $\dfrac{\sin 3A + \sin 4A + \sin 5A}{\cos 3A + \cos 4A + \cos 5A} = \tan 4A.$

5. Find an expression for all the angles which have the same tangent as a given angle.

If
$$\frac{1 + \tan\theta}{1 - \tan\theta} = 3\frac{1 - \tan\theta}{1 + \tan\theta},$$

prove that
$$\theta = n\pi + \frac{\pi}{12} \text{ or } n\pi + \frac{5\pi}{12},$$

where n is any integer.

6. If A, B, C are the angles of a triangle, prove the formulæ

(i.) $\sin A + \sin B + \sin C = 4 \cos\frac{A}{2} \cos\frac{B}{2} \cos\frac{C}{2}.$

(ii.) $\cot\frac{A}{2} + \cot\frac{B}{2} + \cot\frac{C}{2} = \cot\frac{A}{2} \cot\frac{B}{2} \cot\frac{C}{2}.$

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7. A tower stands on a horizontal plane, and it is observed that at a position on the plane, distant 237 feet from the foot of the tower, the angle of elevation of the top of the tower is 37° 15′ 15″; find, to two places of decimals, the height of the tower.

8. Prove that, for a triangle ABC,

$$\sin\frac{A}{2} = \left\{\frac{(s-b)(s-c)}{bc}\right\}^{\frac{1}{2}}.$$

Having given that

$$a = 145, \quad b = 235, \quad c = 200,$$

calculate the angle A.

9. Find expressions for the radii of the inscribed and escribed circles of a triangle ABC.

If r, r_1, r_2, r_3, represent these radii, prove that

$$\frac{rr_1}{r_2r_3} = \tan^2\frac{A}{2}.$$

10. A spectator on one side of a straight road is looking in the direction perpendicular to the direction of the road, and observes that a telegraph post is exactly opposite and that α is the angular elevation of the top of this post. He then turns round and observes that β is the angular elevation of the next post. Assuming that the height of each post is the same, and that c is the distance between the posts, prove that the height of each is equal to

$$c \tan\alpha \tan\beta \div \sqrt{\tan^2\alpha - \tan^2\beta}.$$

11. Two cylindrical vessels are filled with water; the radius of one vessel is six inches and its height one foot, and the radius of the other is eight inches and its height one foot and a half; find the radius of a cylindrical vessel eleven inches in height which will just contain the water in the two vessels.

12. Having given that the length of each edge of a regular tetrahedron is four inches, determine, to three places of decimals, the number of square inches in the total surface of the tetrahedron.

Also find the number of cubic inches in the volume of the tetrahedron.

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IV. STATICS AND DYNAMICS.

1. Enunciate and prove the proposition known as the "Polygon of Forces."

OA, OB, OC are three straight lines inclined at angles of 120° to one another; a force of $3P$ acts from A to O, a force of $4P$ from O to B and a force of $5P$ from O to C. Show by a carefully drawn figure how you would graphically determine the magnitude and direction of the resultant of the three forces.

2. B, C are two smooth rings fixed in space at a distance apart = 25 inches, B being 9 inches, and C 16 inches above the ground. A string $ABCD$ passes through the rings and supports equal weights W, W at its extremities A, D. Find the resultant pressures of the string upon the rings.

3. Two parallel forces $5P$, $7P$ act at points A, B respectively. State clearly the position of the line of action and the magnitude of their resultant (i.) when the forces are like, and (ii.) when unlike.

A horizontal bar AB, 7 feet long, is supported at its extremities, and a man of 150 lbs. weight hangs from it by his hands, one being 1 foot from A, the other 3 feet from B. Find the pressures on the supports due to the weight of the man.

4. Three forces (not parallel) acting in one plane upon a rigid body are in equilibrium. Show that if any triangle be formed by drawing straight lines perpendicular to the directions of the forces, its sides will be proportional to the forces.

A triangle ABC (whose weight may be neglected) rests in a vertical plane with the middle points of the sides AB, AC in contact with two smooth pegs, the line joining them being horizontal and parallel to the base BC. Determine graphically, or otherwise, the point in BC where a weight W may be placed without disturbing the equilibrium; and if W = 10 lbs., and AB, AC and BC be 4, 5 and 6 feet respectively, find the pressures on the pegs.

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5. Show that if a body be placed on a horizontal plane it will stand or fall according as the vertical line through its centre of gravity falls within or without the base. Explain what is meant by the "base."

Why does a bicycle rider when on the point of falling over to one side steer his machine to that side?

6. Distinguish between the three kinds of levers and give examples of each.

Two levers OA, OB of lengths 3 and 4 feet respectively can turn in a vertical plane about a common fulcrum O, and their middle points are connected by a string whose length is $2\frac{1}{2}$ feet. Find the least force which applied at A will keep OB horizontal with a weight of 12 lbs. suspended from B. Find also the tension of the string.

7. "Suppose mud composed of coarser particles to fall at the rate of two feet per hour, and these to be discharged into that part of the Gulf-stream which preserves a mean velocity of three miles an hour for a distance of two thousand miles; in twenty-eight days these particles will be carried x miles, and will have fallen only to a depth of y fathoms." (Lyell's *Principles of Geology.*)

Find the numbers given for x and y in this passage.

8. Assuming that the space described from rest in time t by a particle moving with uniform acceleration a is equal to $\frac{1}{2}at^2$, deduce the corresponding expression for the case when the particle has an initial velocity u.

If the space described in the fifth and sixth seconds from rest be 25 feet, find the acceleration.

9. Distinguish between (i.) acceleration and accelerating force, and (ii.) the mass and weight of a body; and investigate an expression for the acceleration of a particle sliding down a smooth inclined plane under the action of gravity.

Two particles start simultaneously from rest, the one down an inclined plane AC of length 25 feet, the other down a plane BC of length 70 feet, the heights of A, B above the horizontal plane through C being 7 and 56 feet respectively. Find which particle will arrive at C first, and when at C how far it will be from the other particle.

10. Two bodies whose masses are P, Q are connected by a string which passes round a smooth pulley; find the acceleration.

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Show that, if $\frac{3}{8}W$ be added to one of the weights in question (2), and $\frac{3}{7}W$ be taken from the other, the pressures on the rings will be unaltered.

11. A particle is projected at an angle of 45° to the horizon with the velocity it would acquire in falling freely for one second; show in a figure the position of the particle at the end of the first, second, and third seconds of the motion; and taking any unit of length to represent the initial velocity, mark against each straight line of your diagram the measure of its length.

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FURTHER EXAMINATION.

V. PURE MATHEMATICS.

[*Full marks may be obtained for about* three-fifths *of this paper. No Candidate must attempt more than* ten *questions. Great importance will be attached to accuracy.*]

1. A moveable circle with constant radius cuts the two fixed straight lines APR, AQS in P, Q, R, and S, prove that for all positions of the circle the sum of the arcs RPQ and PQS is constant.

2. Factorise the expression

$$1+y(1+x)^2(1+xy).$$

3. Solve the equation

$$x^2+px+q=0.$$

If the roots of $x^2+Px+q=0$ be α and β, and the roots of $x^2+px+Q=0$ be γ and δ, find the roots of $x^2+px+q=0$ in terms of α, β, γ, and δ.

4. Find the number of combinations of n things r together.

There are $2n$ letters, of which 2 are a, 2 are b, and so on: find how many different algebraical products can be formed, in each of which the sum of the indices of the letters is 3.

5. Prove that the sum of the squares of the coefficients of $(1+x)^n$ is

$$\frac{|\underline{2n}}{|\underline{n}\,|\underline{n}},$$

and find the sum of the products taken two and two together.

6. Expand $\log_e(1+x)$ in ascending powers of x.

Prove that the coefficient of x^n in the expansion of

$$\log_e(1+x+x^2\ldots+x^{m-1})$$

is either

$$-\frac{m-1}{n} \quad \text{or} \quad \frac{1}{n}$$

according as n is, or is not, a multiple of m.

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7. Find the third side of a triangle in terms of the two other sides and the included angle.

Apply the result, or any other method, to prove that if α and β be each less than π, and x be positive,

$$\sqrt{1-2x\cos\alpha+x^2}+\sqrt{1-2x\cos(\beta-\alpha)+x^2}>2\sin\frac{\beta}{2}.$$

8. Find the radii of the three escribed circles of a triangle.

If these circles be discs, fixed in position, and be enveloped by a string drawn tight round them, prove that the sum of the straight portions of the string will be equal to twice the perimeter of the triangle.

9. Find the value of

$$\sin\alpha+x\sin 2\alpha+x^2\sin 3\alpha \textit{ ad infin.}, \ x \text{ being} < 1.$$

Prove that

$$\sin\alpha+x\sin 2\alpha+x^2\sin 3\alpha+\text{etc.} = \sin\alpha+y\sin 2\alpha+y^2\sin 3\alpha+\text{etc.}$$

if x and y are proper fractions satisfying the condition $x+y=2\cos\alpha$.

10. From the given point O, any line OP is drawn meeting a given straight line AB in P, and through P, a line PQ is drawn at a given angle α to OP. Prove that for all directions of OP the line PQ will be a tangent to a certain parabola and find the focus and directrix of this parabola.

11. In the hyperbola prove that $SP-HP$ is equal to the transverse axis.

If circles be described with centres S and H, the difference of whose radii is equal to the transverse axis, and from P any point on the branch of the hyperbola nearer to the larger circle, PS and PH be drawn and produced to meet the circles in Q and R respectively, prove that the locus of the intersection of the tangents to the circles at Q and R will be a straight line.

Enunciate and prove the corresponding proposition when P is on the other branch.

12. Prove that $Ax+By=C$ is the equation of a straight line.

Interpret the constants when

(i.) $A=1$. (ii.) $B=1$. (iii.) $C=1$. (iv.) $A^2+B^2=1$.

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13. Find the intercepts on the axes made by the normal at any point x', y' on the ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1.$$

Hence prove that if M and N be the feet of the perpendiculars upon the axes from any point P of an ellipse, the line MN will be always normal to a concentric and similar ellipse.

14. Find the equations of the two straight lines conjugate to the coordinate axes of x and y respectively in the curve

$$Ax^2+2Bxy+Cy^2=1.$$

Find the condition that these straight lines should coincide, and interpret the result.

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VI. MECHANICS.

[*Full marks may be obtained for about* two-thirds *of this paper. Great importance is attached to accuracy.* N.B.—*g may be taken* = 32.]

1. Forces 1, 2, 4, 4 lbs. respectively, act along the sides AB, BC, CD, DA of a square. Find by graphical construction the magnitude and direction, and point of application in the line BC, of the force which would balance the system.

2. $ABCD$ is a rhombus formed by four weightless rods loosely jointed together, and the figure is stiffened by a weightless rod, of half the length of each, jointed to the middle points of AB, AD. If this frame is suspended from A, and a weight of 100 lbs. attached to it at C, find by graphical construction the thrust of the cross rod.

3. A, B are two fixed pegs, B being at a higher level than A, and a heavy rod rests on B and passes under A; show graphically that, the angle of friction between the rod and the pegs being the same in both, the rod will rest in any position in which its centre of gravity is beyond B, provided the inclination of AB to the horizon is less than the angle of friction; and for any greater inclination determine graphically the limiting distance of the centre of gravity beyond B consistent with equilibrium.

Verify your result from the equations of equilibrium.

4. Find the distance of the centre of gravity of a number of heavy particles in the same straight line from a fixed point in the line.

How many like coins, having diameters 20 times the thickness, can be piled on a table so that their centres may be in a straight line inclined at an angle of 45° to the horizon?

5. Roberval's balance consists of a parallelogram $AA'B'B$, loosely jointed at the angles, whose opposite sides AA', BB', are moveable about smooth pivots through their middle points fixed in a vertical line, and with

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arms CD, $C'D'$ rigidly attached at right angles to AB, $A'B'$ respectively. Prove (by the principle of work or otherwise) that a weight suspended from any point in CD will balance an equal weight suspended from any point in $C'D'$, in any position of the system. Determine the horizontal stresses at the angular points of the parallelogram. Can you determine the vertical stresses at the same points?

6. The trail of smoke from a steamer on a course due north is observed to extend in the direction E.S.E., while that from another, on a course due south, with the same uniform speed, is observed to be N.N.E.; determine the speed and direction of the wind.

7. A steamer starts from a pier, steering eastward with uniform acceleration, the wind blowing steadily from the south. Show that the line of trail of its smoke is an arc of a parabola, with a fixed directrix and constant latus rectum.

8. Prove that the velocity at any point in the path of a projectile (*in vacuo*) is that due to its distance from the directrix.

If AC, BC bisect the angles between AB and the verticals through A, B respectively, and if F is the foot of the perpendicular from C on AB, the least velocity of projection which will carry a projectile from A to B is that due to a height equal to AF.

9. Show that, if a point moves in a curve with constant speed, its acceleration at any point is in the normal at that point, and determine the acceleration when the curve is a circle.

A mass of 1 lb. is suspended by a string 2 feet long in a railway carriage. Show that when the train is moving round a curve, whose radius is 22 chains, at the rate of 30 miles an hour, the tension of the string is increased by about $\frac{1}{72}$ oz., and the horizontal displacement of the weight is 1 in. nearly.

10. Define a *foot-pound* and a *horse-power*.

A locomotive engine, which can work up to 100 horse-power, is attached to a train, whose mass (including the locomotive itself) is 100 tons; assuming the total resistance to be constant and equivalent to 10 lbs. weight per ton, find the greatest speed of the train in miles per hour. When travelling at this speed the steam is shut off; find the distance and the time in which the train would be reduced to rest by the resistance alone.

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11. Prove that the kinetic energy of a system of two masses m_1, m_2, moving with the speeds u_1, u_2 in the same straight line, is equal to

$$\tfrac{1}{2}(m_1+m_2)\bar{u}^2+\tfrac{1}{2}\frac{m_1m_2}{m_1+m_2}(u_1-u_2)^2,$$

where $\bar{u}$ is the speed of their centre of mass; and hence, that the kinetic energy lost by their collision is equal to

$$\tfrac{1}{2}\cdot\frac{(1-e^2)m_1m_2}{m_1+m_2}(u_1-u_2)^2$$

where e is the coefficient of resilience.

Account generally for the energy said to be "lost."

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MATHEMATICAL EXAMINATION PAPERS

FOR ADMISSION INTO

Royal Military Academy, Woolwich,

NOVEMBER, 1893.

OBLIGATORY EXAMINATION.

I. EUCLID (BOOKS I.—IV. AND VI.).

[*Ordinary abbreviations may be employed; but the method of proof must be geometrical. Proofs other than Euclid's must not violate Euclid's sequence of propositions. Great importance will be attached to accuracy.*]

1. Any two sides of a triangle are together greater than the third side.

ABC is any rectilineal angle less than the angle of an equilateral triangle, and D and E are two points within it; find the points F and G in AB and BC respectively, such that the sum of the lines DF, FG, GE has the least possible value.

2. Equal triangles on the same base and on the same side of it are between the same parallels.

Use this proposition to show that the straight line joining the middle points of two sides of a triangle is parallel to the third side.

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3. Describe a parallelogram equal to a given rectilineal figure, and having an angle equal to a given rectilineal angle.

4. If a straight line be divided into two equal, and also into two unequal parts, the squares on the two unequal parts are together double of the square on half the line and of the square on the line between the points of section.

Express the proposition as an algebraical formula.

5. Draw a straight line from a given point, either without or in the circumference, which shall touch a given circle.

From a given point P without the circle ABC, draw a straight line PAB cutting the circle in A and B so that AB may be equal to a given straight line D.

6. Distinguish between the angle *of* a segment and the angle *in* a segment and define *similar segments of circles.*

$ABCD$ ... is a straight line, on parts of which AB, AC, AD, ... similar segments of circles AFB, AGC, AHD, ... are described; show that (1) all these circles have a common tangent at A, (2) any straight line $AFGH$ ... drawn from A and cutting the circles in F, G, H, ... cuts off similar segments.

7. On a given straight line describe a segment of a circle containing an angle equal to a given rectilineal angle.

Given the altitude, the vertical angle, and the perimeter of a triangle, construct it.

8. Describe a circle about a given triangle.

Point out and prove any facts concerning the opposite angles or sides of a quadrilateral (1) inscribed in, (2) described about, a circle.

9. When is the first of four magnitudes said to have the same ratio to the second that the third has to the fourth?

Illustrate this definition by giving Euclid's proof of the proposition that triangles of the same altitude are to one another as their bases.

10. Parallelograms which are equiangular to one another have to one another the ratio which is compounded of the ratios of their sides.

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11. In any right-angled triangle, any rectilineal figure described on the side subtending the right angle is equal to the similar and similarly described figures on the sides containing the right angle.

If BFC, CDA, AEB be equilateral triangles described externally on the sides of a triangle ABC right-angled at A; and if AG be drawn perpendicular to BC to meet BC in G; show that the triangles BFG, CFG are respectively equal to the triangles AEB, CDA.

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II. ALGEBRA.

(Up to and including the Binomial Theorem, the theory and use of Logarithms.)

[N.B.—*Great importance will be attached to accuracy.*]

1. Divide $(x+y)^4+(x^2-y^2)^2+(x-y)^4$ by $3x^2+y^2$.

2. Resolve each of the following into three real factors:

$$4x^3-23x^2+28x, \quad y^4+11y^2-180, \quad a^6+27b^6.$$

3. Find the Highest Common Factor of

$$5x^4-16x^3+20x+7 \quad \text{and} \quad 30x^3-71x^2+49.$$

4. Simplify

(i.) $\dfrac{1}{x^2-4x+3}-\dfrac{4}{x^2+2x-15}+\dfrac{3}{x^2+4x-5}$;

(ii.) $\left\{\dfrac{a^3}{b^3}-\dfrac{b^3}{a^3}-3\left(\dfrac{a^2}{b^2}+\dfrac{b^2}{a^2}\right)+5\right\}\div\left(\dfrac{a}{b}-1-\dfrac{b}{a}\right)^2$;

(iii.) $\left\{\dfrac{\sqrt{3}}{\sqrt{5}+\sqrt{2}}-\dfrac{\sqrt{3}}{\sqrt{500}-\sqrt{200}}\right\}\div\left\{\dfrac{5}{\sqrt{15}+\sqrt{6}}-\dfrac{1}{\sqrt{60}-\sqrt{24}}\right\}$.

5. Find the square root of

(i.) $4x^4+12x^3-11x^2-30x+25$;

(ii.) $41-\sqrt{720}$.

6. Solve the equations:

(i.) $ax-by=a^2. \quad bx-ay=b^2$;

(ii.) $\dfrac{7x}{1-\dfrac{2x-12}{3x-5}}=\dfrac{48}{1-\dfrac{1}{x}}$.

Divide 1 into two fractions such that the sum of their cubes is $\frac{1}{8}$.

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7. When are the roots of $ax^2+bx+c=0$ real?

If x is real, show that $\dfrac{x^2-4x-20}{x-7}$ cannot have a real value between 8 and 12.

8. Show that a ratio of greater inequality is diminished by adding any, the same, quantity to both terms of the ratio.

9. If b is half the Harmonic Mean between a and c, then

$$a^3-b^3+c^3+3abc=(a-b+c)^3.$$

10. Prove by *Mathematical Induction* that, when n is a positive integer, $(3n+1)7^n-1$ is always divisible by 9.

11. When $x=\frac{1}{3}$, find the two greatest terms in the expansion of

$$(1-x)^{-12}.$$

12. Prove that

$$\log_y x \times \log_z y \times \log_x z = 1.$$

Show, with the aid of the logarithmic tables supplied, that approximately

$$200^{\frac{1}{10}}+\log_{10}{\cdot}200149=1.$$

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III. PLANE TRIGONOMETRY AND MENSURATION.

(Including the Solution of Triangles.)

[N.B.—*Great importance will be attached to accuracy.*]

1. If the radius of a circle be 25 feet, find, to three places of decimals, the length of the arc subtending an angle of 3° at the centre.

$$\pi = 3{\cdot}1416.$$

2. Find $\tan\theta$ in terms of $\cos\theta$, and $\tan(\phi+\theta)$ in terms of $\tan\phi$ and $\tan\theta$.

If $\cos\theta = \frac{3}{5}$ and $\phi - \theta = \frac{\pi}{4}$, find $\tan\phi$ and $\tan(\phi+\theta)$.

3. Prove that

$$\sin A + \sin B = 2\sin\frac{(A+B)}{2}\cos\frac{(A-B)}{2},$$

and express in a single term

(i.) $\sin A \sin 2A + 2\cos A \cos 2A$.

(ii.) $\dfrac{1}{\sqrt{2}\cot\frac{\theta}{2} - \operatorname{cosec}\frac{\theta}{2}} + \dfrac{1}{\sqrt{2}\cot\frac{\theta}{2} + \operatorname{cosec}\frac{\theta}{2}}$.

4. If A and B are two angles, each positive, and less than 90° and such that

$$3\sin^2 A + 2\sin^2 B = 1,$$

$$3\sin 2A - 2\sin 2B = 0.$$

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prove that

$$A + 2B = 90^\circ.$$

5. Prove the following identities:

(i.) $2 \operatorname{cosec} 4A + 2 \cot 4A = \cot A - \tan A.$

(ii.) $(1 + \sec\theta + \tan\theta)(1 + \operatorname{cosec}\theta + \cot\theta)$
$= 2(1 + \tan\theta + \cot\theta + \sec\theta + \operatorname{cosec}\theta).$

6. Find the cosines of the angles of a triangle in terms of the sides.

If the sides of a triangle are in arithmetical progression, prove that the middle angle must be less than 60°.

7. If one side of a triangle be equal to m times the difference of the other two, prove that the cosine of half the angle subtended by that side will be m times the sine of half the difference of the remaining angles.

8. If in a triangle $a = 3$, $b = 1$, $C = 53^\circ\ 7'\ 48''$, find c.

9. What is meant by the ambiguous case in the solution of plane triangles and when does it occur?

Prove that the sum of the areas of the two triangles satisfying the given conditions is known, and find it in terms of given quantities.

10. A building on a square base $ABCD$ has the sides of the base AB and CD, parallel to the banks of a river. An observer standing on the bank of the river furthest from the building in the same straight line as DA finds that the side AB subtends at his eye an angle of 45°, and after walking a yards along the bank he finds that DA subtends the angle whose sine is $\frac{1}{3}$.

Prove that the length of each side of the base in yards is $\frac{1}{2}\sqrt{2}a$.

11. A cylindrical boiler, terminated by plane ends, is internally 15 feet long and 4 feet in diameter and is traversed lengthwise by 50 cylindrical fire tubes, each 3 inches in external diameter, determine the volume of water the cylinder could contain, taking π to be $\frac{22}{7}$.

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12. Supposing an ice field to exist round one of the earth's poles, extending 5° from the pole in all directions, find the area of the ice field in square miles, taking the earth's radius to be 4000 miles, cos 5° to be ·996195 and $\pi = \frac{22}{7}$.

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IV. STATICS AND DYNAMICS.

[N.B.—*The acceleration due to gravity is to be taken* = 32.]

1. Enunciate the proposition known as the "Parallelogram of Forces," and assuming it for the direction, prove it for the magnitude, of the resultant.

Four forces in equilibrium acting at a point are represented in magnitude and direction by AB, CD, AD, CB. Show that A, B, C, D must be the angular points of a parallelogram.

2. Given the magnitudes of the forces acting in one plane on a particle and the angles which their directions make with a given straight line in the plane, find the magnitude of their resultant and the angle which its direction makes with the given line.

Forces of 3, 4 and 6 lbs. make angles of 90°, 60°, and 30° respectively with a force of 2 lbs. (the angles being measured in the same direction). Show that the resultant of the four forces is equal to $(8+3\sqrt{3})$ lbs.; and find (with the aid of the logarithmic tables supplied) the angle its direction makes with the force of 2 lbs.

3. Define the centre of gravity of a rigid body, and show how its position may be experimentally determined.

A lamina hangs freely in a vertical plane with a point A fixed, and in this position a horizontal line BC is drawn upon it; it next hangs with B fixed and a horizontal line through A is drawn cutting BC in C; show that, if C be fixed, the lamina will rest with the line joining A, B horizontal.

4. State clearly the conditions of equilibrium of a system of forces acting in one plane upon a rigid body.

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A square board $ABCD$ is placed upon a smooth horizontal table and a given force P acts from E, the middle point of AB, towards F, the middle point of CD. Find the magnitudes of three forces X, Y, Z which, acting along BC, CA, AB respectively, will make equilibrium with P.

5. Investigate the relation between the power and the weight in that system of pulleys in which the same string passes round all the pulleys.

Draw a diagram of the case in which there are three pulleys at each block.

6. Define *velocity*, and show how it is measured (i.) when uniform and (ii.) when variable.

A train is moving at the rate of 40 miles an hour, what would be the measure of its velocity if a *yard* and a *second* were the units of space and time?

7. Investigate an expression for the space described from rest in time t by a point moving with acceleration a.

What would be the expression for the space if the point started with a velocity u?

A particle is projected vertically upwards with a velocity of 50 feet per second, when will it be at a height of 25 feet above the point of projection? Also, where will it be when it is moving with a velocity of 25 feet per second?

8. Explain the meanings of the symbols in the equation $F = Ma$, and having given that the units of space and time are one foot and one second, state the relation implied between the other units involved.

Find the force which acting on a mass of 8 ounces increases its velocity by 4 feet a minute in every second; and state clearly the unit in terms of which your answer is expressed.

9. A string attached to a body of mass P placed on a smooth horizontal table passes over one of the edges and has a body of mass Q attached to its other extremity. Find the acceleration of the system.

If Q be divided into two equal parts one hanging below the other by a separate string, find the tension of this string.

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10. A particle is projected in a given direction with a given velocity, show how to find its position and the direction in which it is moving at the end of a given time.

A stone is thrown from the top of a tower with a velocity of g feet per second in a direction making an angle α with a line drawn vertically upwards through the point of projection; prove that at the end of two seconds the line joining the stone to the point of projection will make an angle of $\frac{1}{2}(\pi+\alpha)$ with the vertical line.

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FURTHER EXAMINATION.

V. PURE MATHEMATICS.

[*Full marks may be obtained for about* three-fifths *of this paper. No Candidate must attempt more than* ten *questions. Great importance will be attached to accuracy.*]

1. Describe a circle touching three given straight lines.

If a circle be inscribed in an isosceles right-angled triangle, show that its diameter is equal to the excess of the sum of the equal sides over the hypotenuse.

2. Under what circumstances will the solution of two simultaneous equations in two unknown quantities depend upon the solution of a quadratic equation?

Two trains are proceeding in the same direction upon the same line of rails, each with uniform speed. The quicker train, which is in front, gains $\frac{1}{n}$ of a mile every minute, and also $\frac{1}{n}$ of a minute every mile upon the slower train. Determine the speed of each train in miles per hour, and show that, n being a positive quantity, the speed of the quicker train cannot be less than 60 miles an hour.

3. Find the sum of n terms of a geometrical progression. Sum to n terms, and also to infinity, the series

$$(1+x^n)^{-\frac{1}{n}}+x(1+x^n)^{-\frac{2}{n}}+x^2(1+x^n)^{-\frac{3}{n}}+\dots.$$

4. State the circumstances under which

$$(1+x)^n$$

is expressible in the form of a convergent infinite series.

Show how to expand

$$(1+yz+zx+xy)^{-1}$$

by the binomial theorem, and show that the coefficient of x^3y^2z is equal to the coefficient of the same term in the development of the product

$$(y-z)^3(z-x)^2(x-y).$$

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5. What is the characteristic of the logarithm of a number? Prove the rule for assigning the characteristic of a given decimal number.

Expand $\log_a x$ in a series proceeding by ascending powers of

$$\frac{x-1}{x+1}.$$

6. One angle of a triangle is given, and the ratio of the sines of the other angles. Show how to find these angles by a formula adapted to logarithmic computation.

Find the angles B, C from the data

$$A = 60^\circ, \quad \frac{\sin B}{\sin C} = 2.$$

7. Express the area of a triangle in terms of its sides. Show that the formula involves Euclid's theorem that two sides of a triangle are together greater than the third side.

Find an expression for the area in terms of the sums of first, second, and third powers of the sides.

If the sum of the sides is 12 feet, and the sum of the squares on the sides is 50 square feet, and the sum of the cubes on the sides is 216 cubic feet, find the area of the triangle.

8. Find expressions for the curved surface and volume of a right circular cone.

A conical tent is five feet high. Find the radius of its base so that the number of square feet of canvas may be equal to the number of cubic feet of space inside the tent.

9. Show how to transform from rectangular coordinates to oblique, and from oblique to rectangular.

The equation of a straight line referred to axes inclined at 30° is

$$y = 2x + 1.$$

Find its equation referred to axes inclined at 45°, the origin and axis of x remaining unchanged.

10. Define an ellipse, and prove geometrically that the sum of the focal radii of any point upon it is equal to the major axis.

11. Points (1, 0), (2, 0), are taken on the axis of x, the axes being rectangular. On the line connecting the points an equilateral triangle is described so that the coordinates of its vertex are both positive. Find the equations of the circles described upon its sides as diameters.

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12. Find the equation to the tangent at any point of an ellipse, and the lengths of its intercepts upon the coordinate axes.

In an ellipse of eccentricity $\tan\beta$, a focal chord is inclined at an angle α to the major axis; show that the tangents at the extremities of the chord include an angle

$$\tan^{-1}(\tan 2\beta \sin \alpha).$$

13. If a small portion of a hyperbola be given, show how to verify its hyperbolic nature geometrically.

If two sides of a triangle be given in position, and its perimeter given in magnitude, show that the locus of the point which divides the base in a given ratio is a hyperbola.

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VI. MECHANICS.

[*Full marks may be obtained for about* two-thirds *of this paper. Great importance is attached to accuracy.* N.B.—*g may be taken* = 32].

1. Forces are represented in magnitude, direction and position, by the sides of a polygon taken in order; prove that they are equivalent to a single couple, and that the magnitude of the couple is proportional to the area of the polygon.

2. A uniform beam, 12 feet in length, has a fixed hinge at one end, and is supported by a cord, 13 feet long, attached to the other end and to a fixed point situated 20 feet vertically above the hinge. Find the tension of the cord, assuming the weight of the beam to be 140 lbs.

3. A uniform beam AB, 20 feet long, is supported by props at C and D, two points at distances 4 feet from A and 6 feet from B, respectively. If a load of 1 ton be placed at each extremity of the beam, calculate the magnitude of the moment which tends to break the beam at the middle point of CD: assuming the weight of the beam to be 2 tons.

4. A ladder is placed with one end on a rough horizontal plane and the other against a rough vertical wall; find, by geometrical construction, or otherwise, the limiting position of equilibrium, being given the coefficients of friction and the centre of gravity of the ladder.

If an additional load be placed at any point on the ladder, in this limiting position, find whether the equilibrium will be disturbed or not.

5. Three forces acting on a rigid system are in equilibrium, prove that their directions lie in the same plane, and either pass through a common point or are parallel.

A uniform cubical block is sustained on a rough inclined plane by a rope, which is parallel to the plane and is attached to the middle point of the upper edge of the cube, which is horizontal. The rope lies in the vertical plane, which contains the centre of the cube and is perpendicular to the inclined plane. Determine the greatest inclination of the plane consistent with equilibrium.

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6. Two particles start simultaneously from different points, in given directions, with uniform velocities. Show, by geometrical construction, how to find their relative distance at the end of any time; and determine when this distance is least.

7. Prove that the path of a projectile (*in vacuo*) is a parabola, and that the velocity at any point is equal to that due, under the action of gravity, to the vertical distance of the point from the directrix.

A train is moving with a velocity of 30 miles an hour when a ball is dropped from the roof of one of the carriages 10 feet above the earth. Show how to find the focus and directrix of the parabolic path described by the ball, relatively to the earth.

8. Enunciate accurately Newton's Laws of Motion.

A train is moving on a horizontal railroad. Assuming the weight of the train (exclusive of the engine) to be 120 tons, and the resistance arising from friction, etc., to be 10 lbs. per ton, find the tension of the couplings of the carriage which is attached to the engine, (1) when the velocity of the train is uniform, (2) when it is moving with an acceleration of 4 feet per second, per second.

9. Two imperfectly elastic spherical bodies, whose centres are moving in the same straight line with given velocities, impinge on each other: show how to find their velocities immediately after the impact.

Two spheres meet directly with equal and opposite velocities, find the ratio of their masses in order that one of them should be brought to rest by the collision, (1) when perfectly elastic, (2) for coefficient of resilience e.

10. A ball moving with a velocity of 500 feet per second has its velocity reduced by 50 feet after penetrating 1 inch into a plank. Find how far it will penetrate into the plank before being stopped, assuming the resistance of the plank to be uniform.

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MATHEMATICAL EXAMINATION PAPERS

FOR ADMISSION INTO

Royal Military Academy, Woolwich,

JUNE, 1894.

OBLIGATORY EXAMINATION.

I. EUCLID (BOOKS I.—IV. AND VI.).

Ordinary abbreviations may be employed; but the method of proof must be geometrical. Proofs other than Euclid's must not violate Euclid's sequence of propositions. Great importance will be attached to accuracy.]

1. The opposite sides and angles of a parallelogram are equal to one another.

2. The complements of the parallelograms which are about the diameter of any parallelogram, are equal to one another.

A point E is taken in the side AB of the parallelogram $ABCD$ and ED and EC are joined; prove that, if the line HK parallel to AB cuts ED and EC in F and G respectively, the parallelogram $AHKB$ will be double of either of the triangles EDG or ECF.

3. If a straight line be divided into two equal parts and also into two unequal parts, the rectangle contained by the unequal parts, together with the square on the line between the points of section, is equal to the square on half the line.

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4. Divide a given straight line into two parts, so that the rectangle contained by the whole and one of the parts may be equal to the square on the other part.

The line AB is bisected in C and produced to D, so that the square on CD is equal to the sum of the squares upon AB and BC, prove that the rectangle AD, DB is equal to the square on AB.

Express the ratio of BD to AB algebraically.

5. State, *without proof*, the difference between the square on one side of a triangle and the sum of the squares of the two remaining sides.

A point P is taken on the circumference of the circle APB, whose centre is O, and with P as centre, another circle, QAB, is described, cutting the former in A and B. If QM be drawn from Q, any point on QAB outside of APB, perpendicular to AB produced, prove that twice the rectangle of QM and OP is equal to the difference of the squares on OQ and OA.

6. The angle at the centre of a circle is double of the angle at the circumference on the same base, that is, on the same arc.

7. The opposite angles of any quadrilateral figure inscribed in a circle are together equal to two right angles.

8. Describe a circle about a given triangle.

The straight line AB of given length moves so that its extremities are respectively upon the two fixed straight lines OC and OD meeting at O. Prove that the centre of the circle circumscribing the triangle OAB lies upon the circumference of a circle whose centre is O.

9. Inscribe an equilateral and equiangular pentagon in a given circle.

10. If the vertical angle of a triangle be bisected by a straight line which also cuts the base, the segments of the base shall have the same ratio which the other sides of the triangle have to one another, and if the segments of the base have the same ratio which the other sides of the triangle have to one another, the straight line drawn from the vertex to the point of section shall bisect the vertical angle.

11. If four straight lines be proportionals, the rectangle contained by the extremes is equal to the rectangle contained by the means; and if the rectangle contained by the extremes be equal to the rectangle contained by the means, the four straight lines are proportionals.

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12. The diameter BCA of the circle $APQB$ whose centre is C, is produced through A to O, and from O the line OPQ is drawn, cutting the circle in P and Q, prove that the triangles OPA and OQB are similar, and prove that if the circle circumscribing the triangle PCQ meets OC in D then

(1) The point D will be fixed for all directions of the line OPQ;

(2) The ratios $OA : AD$ and $OC : CP$ will be equal.

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II. ARITHMETIC.

[N.B.—*The working as well as the answers must be shown.*]

1. Find the value of

$$\frac{5\frac{1}{7}-3\frac{5}{14}}{2\frac{1}{2}+3\frac{3}{4}}\times 79\tfrac{1}{2}.$$

2. Divide ·0468 by 29·25.

3. What is the interest on £775 for $3\frac{1}{2}$ years at $2\frac{3}{4}$ per cent. per annum?

4. Find the value of £1·69375.

5. What fraction of an acre is 28 poles?

6. Find by Practice the value of 13 lbs. 7 ozs. 5 dwts. 8 grs. of silver at 3*s.* 9*d.* per oz.

7. Find the least common multiple of 132, 165, 220.

8. A cube of metal, each edge of which measures $\frac{5}{8}$ of an inch, weighs ·625 lb. What is the length of each edge of a cube of the same metal which weighs 40 lbs.?

9. If 9 men do $\frac{3}{8}$ of a piece of work in 14 days, working 10 hours a day, how many extra men must be employed to finish the work in 5 days more if all of them are to work only 8 hours a day?

10. Prove that the difference of the squares of any two odd numbers is divisible by the double of their sum, and that, if the numbers are consecutive odd numbers, the difference of their squares is a multiple of 8. (N.B.—*You may take any two odd numbers which you like to select, show that the propositions are true for those numbers, and then extend your reasoning to all odd numbers.*)

11. A's money is to B's money as 3 : 7; and if B pays A £345, the proportion will be 3 : 5. How much money has each?

12. If 9 men and 6 boys can do in 2 days what 5 men and 7 boys could do in 3 days, in what time could 2 men and 5 boys do the same?

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13. A dealer has two sorts of tea, one of which he could sell at 1*s.* 8*d.* per lb. and make 25 per cent. on his outlay, the other at 2*s.* 3*d.* and make $12\frac{1}{2}$ per cent. What profit per cent. will he make if he mixes them in equal quantities and sells the mixture at 1*s.* 11*d.* per lb.?

14. Goods are imported from abroad at an expense equal to 35 per cent. of the cost of production; and the importer makes 15 per cent. on his whole outlay by selling them to a tradesman at £7. 15*s.* 3*d.* per ton. Find the cost per ton of production.

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III. ALGEBRA.

(Up to and including the Binomial Theorem, the theory and use of Logarithms.)

[N.B.—*Great importance will be attached to accuracy.*]

1. Divide $x^5+x^4+4x^3+21x^2+23x-40$ by x^2+4x+5.

2. Resolve into factors the expressions:

$$x^3-8y^3, \quad x^2-24xy+128y^2, \quad \text{and} \quad x^4+x^2y^2+y^4.$$

3. Find the Highest Common Factor of the expressions:

$$x^4-8x^3+13x^2-30x+8 \quad \text{and} \quad x^4-4x^3-11x^2-50x+16.$$

4. Prove that $(b-c)^5+(c-a)^5+(a-b)^5$
is divisible by $(b-c)(c-a)(a-b)$,
and find the other factor.

5. Find the square roots of the expressions

(i.) $n(n+1)(n+2)(n+3)+1$.

(ii.) $24+\sqrt{572}$.

6. Solve the equations

(i.) $\dfrac{x+a}{x-c}+\dfrac{x+c}{x-a}=2$;

(ii.) $x^2+3x+4\sqrt{x^2+3x-3}=48$.

7. If α and β are the roots of the equation $x^2+px+q=0$, prove that $\alpha+\beta=-p$ and $\alpha\beta=q$, and find, in terms of p and q, the equation of which the roots are

$$\alpha+2\beta \quad \text{and} \quad \beta+2\alpha.$$

8. At present the ratio of B's age to A's age is the ratio 5 : 2, but in 30 years' time the ratio will be 35 : 23; find their ages.

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9. Find the least possible positive value of the expression

$$4x+\frac{9}{x},$$

and the greatest possible value of the expression

$$\frac{x+2}{4x^2+16x+25}.$$

10. If the expression $x(1-5x+6x^2)^{-1}$ be expanded in powers of x, prove that the coefficient of x^r is 3^r-2^r.

11. Write down an expression for the number of combinations of n things taken r together, and, if ${}_nC_r$ represent this expression, prove, by general reasoning or otherwise, that

$${}_nC_r = {}_nC_{n-r}, \quad \text{and} \quad {}_{n+1}C_r = {}_nC_r + {}_nC_{r-1}.$$

12. Having given

$$\log_{10}5 = \cdot 6989700, \quad \log_{10}7 = \cdot 8450980, \quad \text{and} \quad \log_{10}11 = 1\cdot 0413927,$$

find the logarithms, to the base 10, of 385 and $\frac{176}{121}$, and solve approximately, to two places of decimals, the equation

$$5^x \cdot 7^{x-1} = 11^{x+2}.$$

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IV. PLANE TRIGONOMETRY AND MENSURATION.

(Including the Solution of Triangles.)

[N.B.—*Great importance will be attached to accuracy.*]

[$\pi = \frac{22}{7}$.]

1. Explain what is meant by the *circular measure* of an angle, and find the circular measure of an angle of x degrees.

A wire AB, 1 foot in length, is bent so as to form an arc of a circle whose diameter is four inches; find the angle subtended at the centre of the circle by the chord AB.

2. Express the Trigonometrical ratios of an angle in terms of the secant, the angle being less than a right angle.

If in a triangle ABC,

$$CA = CB = 2, \quad \text{and} \quad AB = 3;$$

find the value of

$$(\sin A - \cos A)(\sec A + \operatorname{cosec} A).$$

3. Determine the values of the Trigonometrical ratios for an angle of 60°, and for an angle of 30°.

A ladder rests against a vertical wall at an angle of 60° with the horizon, and when the foot is drawn back 18 feet further from the wall the inclination to the horizon is found to be 30°. Find the length of the ladder.

4. Prove that $\cos(90° + A) = -\sin A$ for the case when A is less than two right angles.

Write down the values of $\sin 225°$, $\cos 210°$, $\tan 315°$, and $\operatorname{cosec} 420°$.

5 Show geometrically that $\cos(A + B) = \cos A \cos B - \sin A \sin B$, where A, B are two positive angles whose sum is less than a right angle.

Find the value of $\cos 75°$.

6. Express $\sin 3A$ and $\cos 3A$ in terms of $\sin A$ and $\cos A$, and show that

$$\cos 3A + \sin 3A = (\cos A - \sin A)(1 + 2 \sin 2A).$$

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7. Prove the following identities:

(i.) $(x \tan \alpha + y \cot \alpha)(x \cot \alpha + y \tan \alpha) = (x+y)^2 + 4xy \cot^2 2\alpha$.

(ii.) $\cos \beta \cos(2\alpha - \beta) = \cos^2 \alpha - \sin^2(\alpha - \beta)$.

(iii.) $\cos \alpha + \cos 3\alpha + \cos 5\alpha + \cos 7\alpha = \frac{1}{2} \sin 8\alpha \operatorname{cosec} \alpha$.

8. In any triangle ABC find $\tan \frac{1}{2}A$ in terms of the sides a, b, c.

Find the angles of the triangle whose sides are proportional to 3, 5, 7.

9. Show how to solve a triangle when two sides and (i.) an angle opposite to one of the sides, and (ii.) the included angle, are given.

From a boat at sea the angle subtended by the line joining two fixed objects A, B on land is observed to be α; after sailing x yards directly towards A the angle subtended by AB is found to be β, and then after sailing y yards directly towards B, the angle is found to be γ. Find an expression for the distance AB.

10. In a triangle ABC, $a = 35$, $b = 43$, and $C = 75° \, 10' \, 40''$, find the angles A and B.

11. Find the area enclosed by 200 hurdles placed so as to form a regular polygon of 200 sides, the length of each hurdle being 6 feet.

12. A leaden sphere one inch in diameter is beaten out into a circular sheet of uniform thickness $= \frac{1}{100}$th inch. Find the radius of the sheet.

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V. STATICS AND DYNAMICS.

[*You may assume that* $g = 32$ *feet per second per second.*]

1. State accurately the principle known as that of the *Triangle of Forces.*

A particle is acted upon by three forces of given magnitudes; show how these forces must be arranged so as if possible to produce equilibrium, when

(i.) the forces have magnitudes represented by the numbers, 6, 11, 18.

(ii.) their magnitudes are represented by 8, 15, 17.

2. Let O be the position of a particle, and OA a right line drawn through O. Find the magnitude and direction of the resultant of forces proportional to 10, 18, 20, 16,
acting on the particle, when their lines of action make with OA angles of
$0°, 30°, 90°, 135°,$
respectively, all measured in the same sense (*i.e.*, all outwards from O, or all inwards towards O).

3. A body whose mass is 520 pounds is placed on a smooth inclined plane, the tangent of whose inclination to the horizon is $\frac{5}{12}$; find the force necessary to sustain the body

(i.) when this force is horizontal;

(ii.) when it acts along the inclined plane.

4. Define a couple, and show that its moment is the same about all points in its plane.

Prove that the sum of the moments of any two forces in the same plane about any point in the plane is equal to the moment of their resultant about the point.

5. Two parallel forces, P, Q, act on a rigid body; find the magnitude and line of action of their resultant (1) when they act in the same direction; (2) when they act in opposite directions.

Two parallel forces of 20 and 25 pounds' weight, of opposite senses, act on a rigid body, the perpendicular distance between their lines of action being 4 inches; find the resultant.

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6. Show how to find the position of the centre of gravity of a given system of particles whose masses are m_1, m_2, m_3, ... occupying given positions.

At each vertex of a triangle is placed a particle whose mass is proportional to the length of the opposite side; show that the centre of gravity is the centre of the inscribed circle.

From a thin uniform circular plate of radius 13 inches is cut out a circular plate of radius 5 inches, the centre of the latter being 4 inches distant from that of the former; find the position of the centre of gravity of the remainder.

7. Describe the screw press, and find the condition for the equilibrium of the effort and resistance ("power" and "weight") applied to it when there is no friction.

8. Define *acceleration*. State clearly what is meant by saying that g is about 32 feet per second per second, and describe any method by which this value of g has been obtained.

Which is the greater acceleration, 15 miles per hour per minute, or $\frac{1}{3}$ feet per second per second?

9. If a particle moves in a right line with constant acceleration, a, having had an initial velocity, u, show that the distance travelled in time t is given by the equation

$$s = ut + \tfrac{1}{2}at^2.$$

A bullet is fired vertically upwards with a velocity of 496 feet per second; 3 seconds afterwards another is fired vertically upwards from the same point with a velocity of 568 feet per second; when and where will they meet?

10. Define an *absolute unit of force*, and, in particular, the *dyne* and the *poundal*.

What force uniformly applied to a mass of 12 pounds will give it an acceleration of 8 feet per second per second? (Express the force both in pounds' weight and in poundals.)

11. From a given point on a horizontal plane is projected a particle with a velocity u at an elevation α; find the range on the plane.

If the particle is projected with a velocity of 500 feet per second at an angle of elevation whose tangent is $\frac{4}{3}$, find the range. Find also the magnitude and direction of the velocity 10 seconds after the time of projection.

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FURTHER EXAMINATION.

VI. PURE MATHEMATICS.

[*Full marks may be obtained for about* two-thirds *of this paper.*]

1. Sum up the conditions under which Euclid says that two triangles are equal.

ABC is a triangle and D a point in BC such that AD bisects the angle A. If O be the centre of a circle which touches AB at A and also passes through D, prove that OD and AC are at right angles and find the magnitude of the angle AOD.

2. Apply the theory of *geometrical progression* to the evaluation of a mixed recurring decimal.

Show that the sum of $\frac{1}{37}$ and the series

$$\frac{38 \times 18}{37^2} + \frac{38 \times 18^2}{37^3} + \frac{38 \times 18^3}{37^4} + \ldots \textit{ad inf.}$$

is unity.

Employ logarithms to evaluate the tenth and twentieth terms of the series.

3. A clock strikes at intervals of one second. Determine the intervals as they appear to men travelling in express trains at 60 miles an hour directly towards and directly away from the clock respectively. *The velocity of sound may be taken as* 1100 *feet per second.*

4. Find the greatest coefficient in the expansion of

$$(x+y)^n.$$

Prove that the greatest coefficients in the expansion of the trinomial

$$(x+y+z)^{3n+1}$$

have the value

$$\frac{(3n+1)!}{(n!)^3(n+1)}.$$

N.B.—$n!$ represents the product $n(n-1)(n-2)\ldots 3 \cdot 2 \cdot 1$.

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5. In any triangle show that, in the usual notation,

$$\cos A + \cos B = \frac{a+b}{c}(1 - \cos C),$$

$$\cos B - \cos A = \frac{a-b}{c}(1 + \cos C).$$

6. Find the volume of a pyramid on a triangular base and deduce that the volume of a sphere is the product of the area of its surface and one-third of its radius.

A sphere is cut by a horizontal plane. If P be a point in the perimeter of the section and A the highest point of the sphere, show that the surface of the sphere above the cutting plane has an area equal to that of a circle of radius AP.

7. In the triangle ABC, sides 4, 3, 5 AB produced is a tangent, BC the axis and AC the tangent at the vertex of a parabola.

Draw the triangle in your book and construct geometrically for (i.) the focus, (ii.) the directrix, (iii.) the point P on the parabola at which AB is tangent, (iv.) the other extremity of the focal chord through P, (v.) the extremities of the latus rectum.

8. In an ellipse QQ' is the chord of contact of tangents from an external point P; CK is the perpendicular on QQ' from the centre C; and PG is the perpendicular to QQ', through P meeting the major axis in G. Prove that the semi-minor axis is a mean proportional to KC and PG.

9. Write down the usual forms of equation to a straight line.

Through a given point P whose coordinates are (p, q) a straight line is drawn intersecting the axes in A and B so that P is the middle point of AB. Determine the equation of AB.

10. Find the general equation of the circle when the coordinate axes are inclined to one another at an angle ω.

The axes being rectangular investigate the condition that must be satisfied by the parameters of the circle in order that it may be possible to find a point or points on the circle at equal perpendicular distances from the axes.

11. In the parabola

$$y^2 = 4ax$$

find the equation to the circle passing through the vertex and the extremities of the latus rectum.

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Find also the coordinates of the points where the circle is intersected by those normals to the parabola which make an angle of 45° with the axis.

12. C is the centre of an ellipse and BB' the minor axis. If S be the focus which (to origin at the centre and coordinate axes coincident with the axes of the ellipse) has a positive abscissa; and $B'S$ be produced to meet the curve in P; show that CP makes an angle ϕ with the major axis such that

$$2e \tan \phi = (1-e^2)^{\frac{3}{2}}.$$

13. Find the equation to the tangent at any point of the hyperbola

$$4xy = a^2 + b^2.$$

If two tangents at right angles intersect at P find the locus of P.

VII. MECHANICS.

[*Full marks may be obtained for about* two-thirds *of this paper. Great importance is attached to accuracy.* N.B.—*g may be taken* = 32.]

1. If three forces acting on a particle are in equilibrium, each force is proportional to the sine of the angle between the other two.

$ABCD$ is a quadrilateral, having the angles at A and D right angles, and $CB = CD$. Forces P, Q, and R, acting along AB, CA, and AD respectively, are in equilibrium. If

$$P = \frac{2}{\sqrt{3}}R,$$

show that $AB = \frac{1}{2}BC$ or $\frac{3}{2}BC$.

2. Prove that the algebraical sum of the moments of a number of coplanar forces acting on a particle about any point in their plane is equal to the moment of their resultant about that point.

P is the orthocentre of a triangle ABC. Forces act along AP, BP, CP, and are proportional to $\sin(A+\theta)$, $\sin(B+\theta)$, $\sin(C+\theta)$. Prove that their resultant passes through the centre of the circumscribed circle. What is the value of θ when the forces are in equilibrium?

3. Two equal uniform rods, AB, BC, each of weight W, are hinged together by a smooth hinge at B, and rest on a smooth cylinder of radius b,

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whose axis is horizontal. The rods touch the cylinder at points distant one-third of their length from B. Find the length of the rods, and prove that the bending moment at the point of contact of either rod with the cylinder is $\frac{2}{3\sqrt{3}} Wb$.

4. A heavy circular wheel is suspended in a vertical plane from a point B in a rough vertical wall, by a smooth weightless wire through its centre O, and rests at right angles to the wall and touching it at the point A. A string is fixed to the rim of the wheel and passes over it on the side away from A. If the inclination (α) of the wire OB to the horizon is less than 2β, where $\tan\beta$ is the coefficient of friction between the wheel and the wall, show that a force, however great, applied along the string will not turn the wheel unless the direction of the string makes with BO produced an angle less than

$$\sin^{-1}\frac{\sin(\alpha-\beta)}{\sin\beta}.$$

(Positive angles are measured from OA to OB.)

5. Find the position of the centre of gravity of a cylindrical bar with circular ends, whose density at any point is proportional to the square of the distance of the point from the end of the bar.

A cone of vertical angle 2α rests with its base on a rough plane inclined at an angle β to the horizon, the coefficient of friction (μ) being greater than $\tan\beta$. A gradually increasing force is applied at the vertex of the cone parallel to the plane and downwards. Show that, when the force is large enough to disturb equilibrium, the cone will tilt over or slide down according as $\tan\beta$ is less or greater than $\frac{4}{3}(\mu - \tan\alpha)$.

6. A particle initially at rest is acted on by a force constant in direction and magnitude. Prove that the kinetic energy of the particle at any time is equal to the work done on it by the force.

A particle moves from rest down a rough plane inclined at an angle 2θ to the horizon, $\tan\theta$ being the coefficient of friction. Prove that in moving over a length s of the plane it acquires the same velocity as in falling freely through a distance $s\tan\theta$.

7. An engine draws a train whose weight (exclusive of the engine) is 100 tons. The power of the engine is such that when running on the level it exerts a pull of 2 tons weight on the front carriage, and the resistance due to friction, etc., is 11·2 lbs. per ton. Show that if the engine draws the same train from rest up an incline of 1 in 300 it will in one minute acquire a velocity slightly exceeding $15\frac{3}{11}$ miles per hour.

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8. A bullet is fired with a velocity of 800 feet per second. Find (*a*) its greatest possible range on the horizontal plane through the point of projection; and (*b*) the height (approximately) to which the bullet ascends when the range is one-tenth of the greatest possible range.

9. A mass of weight W rests on the smooth surface of a horizontal table, also of weight W, and is connected by a weightless string, passing over a smooth pulley at the edge of the table, with a weight $2W$ hanging freely, which is allowed to fall, the string being initially taut. If the table does not move, find the tension of the string, and show that the coefficient of friction between the table and the floor is not less than $\frac{1}{4}$.

10. Two equal imperfectly elastic balls moving in the same straight line impinge directly upon one another. Find the change of kinetic energy produced by the impact.

A billiard ball A moving parallel to one side of the table, strikes another ball B (initially at rest) in the centre. After B has struck the cushion and then struck A again, the velocity of A is three-fourths of its initial velocity, but in the opposite direction. Show that, if e is the coefficient of elasticity between the balls and also between a ball and the cushion,

$$e^3 + e^2 + 3e = 4.$$

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MATHEMATICAL EXAMINATION PAPERS

FOR ADMISSION INTO

Royal Military Academy, Woolwich,

NOVEMBER, 1894.

OBLIGATORY EXAMINATION.

I. EUCLID (BOOKS I.—IV. AND VI.).

[*Ordinary abbreviations may be employed, but the method of proof must be geometrical. Proofs other than Euclid's must not violate Euclid's sequence of propositions. Great importance will be attached to accuracy.*]

1. Define *plane rectilineal angle, circle, gnomon, similar rectilineal figures.*

2. If one side of a triangle be produced, the exterior angle is greater than either of the interior opposite angles.

3. The straight lines which join the extremities of two equal and parallel straight lines towards the same parts are also themselves equal and parallel.

ABCD is a quadrilateral; show that, if the four parallelograms *BCDP*, *CDAQ*, *DABR*, *ABCS* be completed, the four straight lines *AP*, *BQ*, *CR*, *DS* will be equal and parallel.

4. If a straight line be divided into any two parts, the square on the whole line is equal to the squares on the two parts, together with twice the rectangle contained by the two parts.

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Show also that if a straight line be divided into any three parts, the square on the whole line is equal to the squares on the three parts, together with twice the three rectangles whose sides are the three parts taken two and two together.

5. Describe a square equal to a given rectilineal figure.

6. If a straight line drawn through the centre of a circle bisect a straight line in it which does not pass through the centre, it cuts it at right angles; and if it cut it at right angles, it bisects it.

7. Straight lines in a circle which are equally distant from the centre are equal to one another.

8. If a straight line touch a circle, and from the point of contact a straight line be drawn cutting the circle, the angles which this line makes with the line touching the circle are equal to the angles which are in the alternate segments of the circle.

Four circular coins, of different sizes, are placed upon a table so that each one touches two, and only two, of the remaining three; show that the four points of contact lie on a circle.

9. Inscribe a circle in a given triangle.

Prove that the centre of this circle lies inside each of the three circles described on the three sides of the triangle as diameters.

10. Describe a circle about a given equilateral and equiangular pentagon.

11. Triangles which have one angle of the one equal to one angle of the other and the sides about the equal angles reciprocally proportional, are equal to one another.

Two straight lines AOC, BOD intersect in O and the lines AB, CD are drawn. From the greater of the two triangles AOB, COD cut off a part equal to the less by a straight line drawn through the point O.

12. Parallelograms about the diameter of any parallelogram are similar to the whole parallelogram and to one another.

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II. ARITHMETIC.

[N.B.—*The working as well as the answers must be shown.*]

1. Simplify $\frac{2\frac{5}{9} \text{ of } 1\frac{17}{48} - 3}{4\frac{3}{11} - 2\frac{3}{4}} \div 2\frac{3}{21}$.

2. Find the value of $(2{\cdot}37 \times {\cdot}093) \div {\cdot}0005$.

3. A sum of £237. 18*s.* 4*d.*, lent at simple interest, amounts in three years to £270. 0*s.* 8½*d.* What is the rate per cent.?

4. Find the value in cwts. qrs. and lbs. of ·0234375 of 9⅛ tons.

5. What fraction is 28⅞ cubic inches of a cubic foot?

6. Find the value of 13 acres 2 roods 17½ perches at £14. 14*s.* 8*d.* per acre.

7. Find the greatest common measure of 10058, 4982, and 9823.

8. Justify, from first principles, each step of the process of addition of vulgar fractions and deduce the rule for the addition of decimals.

9. What was the cost of goods on which a man lost 20 per cent. by selling them for £64?

10. A clock set right at noon on Tuesday loses at the rate of 192 seconds in 10 hours. What is the true time on the following Friday afternoon when the reading of this clock is 2 hours 36 minutes?

11. Determine, without performing the divisions, the remainders that result from dividing 48909661 by 8, 16, 25, 9, and 11; and give a brief explanation of the *reason* from which you draw your conclusion in the first three cases.

12. If it costs the same amount to keep 4 horses or 9 oxen, and 5 horses can do as much work as 8 oxen, which will it be more profitable to employ —20 horses and 32 oxen, or 12 horses and 48 oxen?

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13. The incomes of two men would be equal if one were increased 7 per cent. and the other diminished $7\frac{1}{2}$ per cent., and the sum of their incomes is £418. 19*s*. What is the income of each?

14. Three trains start from a town *A* at 12.0, 12.5, and 12.10, and travelling each at a uniform rate by 3 different routes of the same length to a town *B*, are observed to pass a signal box at *B* exactly abreast of each other at 12.50. If at 12.20 the sum of the distances traversed by the three trains is 36 miles 7 furlongs, how far from *A* is the signal box at *B*?

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III. ALGEBRA.

(Up to and including the Binomial Theorem; the theory and use of Logarithms.)

[N.B.—*Great importance will be attached to accuracy.*]

1. Divide $x^6 - a^6$ by $x^2 - ax + a^2$.

Show that $x^6 - a^6$ is divisible by $x^2 + px + \frac{1}{2}p^2$, if $p^6 + 27a^6 = 0$.

2. Express

$$\frac{\frac{1}{x} + \frac{1}{y-z}}{\frac{1}{x} - \frac{1}{y-z}}\left\{1 - \frac{y^2 + z^2 - x^2}{2yz}\right\}$$

in its simplest form.

3. Prove, by the method of finding the Greatest Common Measure, that $2x^2 - x - 6$ is a factor of

$$2x^5 + 3x^4 - 32x - 48,$$

and

$$2x^6 - x^5 - 6x^4 + 2x^3 - 3x^2 - 5x + 6.$$

4. Show, by any method, that

$$a^3(b-c) + b^3(c-a) + c^3(a-b)$$

contains $b-c$, $c-a$, $a-b$ as factors, and find the remaining factor.

5. Find the values of x, y, z from the equations

$$x + 2y - z + 4 = 0,$$

$$3x + 4y + z - 1 = 0,$$

$$5x + 6y - 3z + 18 = 0.$$

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6. Simplify the equation

$$\frac{\dfrac{2x-1}{3}-\dfrac{4x^2-1}{x+3}}{x+1-\dfrac{5x^2-9}{3(x-3)}}=\frac{x-3}{x+3}\cdot\frac{10x+1}{2x+3},$$

and solve it.

7. Find the condition that the roots of the equation

$$ax^2+bx+c=0$$

shall be equal.

Determine the values of k for which the equation

$$12(k+2)x^2-12(2k-1)x-38k-11=0$$

will have equal roots.

8. Prove that, if a and l are the first and last terms of an arithmetical progression containing n terms, the sum of the series is

$$\tfrac{1}{2}n(a+l).$$

The sum of 5 terms of an arithmetic series is 10, and the sum of 17 terms is -17; find the series.

9. Expand

$$\frac{1}{\sqrt[3]{1-x}}$$

in a series ascending by powers of x, as far as x^3, by the Binomial Theorem, and write down an expression for the nth term of the series.

10. A and B are two stations on a railway, 90 miles apart. At the same instant one train passes through A towards B and another through B towards A, with different but constant speeds. They pass each other at C, and AC is 10 miles longer than BC; also, the first reaches B half-an-hour before the second reaches A; find their speeds.

11. Define the logarithm of any number to a given base, a.

Find, from a table of common logarithms, the logarithm of 125 to the base $4\frac{1}{2}$. Give, without proof, to four decimal places, the value of the modulus which converts logarithms to base 10 into logarithms to the Napierian base.

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12. If e is the Napierian base, prove, by any method, that

$$\log_e(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots.$$

Hence show that, for any number, n,

$$\log_e n = 2\left\{\frac{n-1}{n+1} + \tfrac{1}{3}\left(\frac{n-1}{n+1}\right)^3 + \tfrac{1}{5}\left(\frac{n-1}{n+1}\right)^5 + \dots\right\}.$$

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IV. PLANE TRIGONOMETRY AND MENSURATION.

(Including the Solution of Triangles.)

[N.B.—*Great importance will be attached to accuracy.* $\pi = \frac{22}{7}$.]

1. Prove that the angle subtended at the centre of a circle by an arc equal to the radius of the circle is of the same magnitude for all circles.

Find this angle in degrees.

2. Write down formulæ for $\sin(A+B)$, $\cos(A-B)$, and $\tan(A+B)$, in terms of the trigonometrical functions of A and B.

Find $\cos 75°$ and $\sin 18°$.

3. Find a value of A, positive and less than 90°, satisfying the equation

$$(\sin A° - \cos A°)(\sec A° - \operatorname{cosec} A°) = 2.$$

4. Prove that for all values of A and B

$$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}.$$

If the sum of two angles be always equal to α, where α is positive and not greater than 180°, prove that the sum of their cosines will be never greater than $2\cos\frac{\alpha}{2}$ and never less than $2\cos^2\frac{\alpha}{2}$.

5. Prove that for all values of A and B

$$\cos B - \cos A = 2\sin\frac{A-B}{2}\sin\frac{A+B}{2}.$$

Find the numerically smallest value of θ, different from zero, satisfying the equation

$$\cos 3\theta - \cos 4\theta = \cos 5\theta - \cos 6\theta.$$

6. Prove that $\sin 33° + \cos 63° = \cos 3°$;
also that, if $\cot A \cot B = 2$,
then $\cos(A-B) = 3\cos(A+B)$.

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7. In the triangle ABC prove that

$$a \cos B + b \cos A = c.$$

$$a \sin B - b \sin A = 0.$$

$$a^2 + b^2 - 2ab \cos C = c^2.$$

8. Given $A = 42°, \quad a = 141, \quad b = 172{\cdot}5,$
find all solutions of the triangle ABC.

9. From the point O the three straight lines OA, OB, OC are drawn in the same plane, of lengths 1, 2, 3 respectively, and with the angles AOB and BOC each equal to 60°. Find the angle ABC correct to one minute.

10. Find the area of the greatest circle which can be cut out of a triangular piece of paper whose sides are 3, 4, 5 feet respectively.

11. A conical extinguisher, whose section through the vertex is an isosceles triangle with vertical angle 30°, is placed over a cylindrical candle whose diameter is one inch, and rests so that the point of contact of the top of the candle with each generating line of the cone bisects that line. Find the whole inside surface of the extinguisher.

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V. STATICS AND DYNAMICS.

[*It may be assumed that* $\pi = \frac{22}{7}$, *and that* $g = 32$, *when a foot and a second are the units of length and time.*]

1. Explain why it is that forces can be completely represented by straight lines.

$ABCD$ is a square; find the resultant of the forces represented by the straight lines AB, AC, and AD.

2. Enunciate and prove the theorems of the triangle of forces and the polygon of forces, and state whether the converses of these theorems are true.

3. A heavy pole, weighing 140 lbs., is carried on the shoulders of two men, one at each end; the centre of gravity of the pole being two feet from one end and five feet from the other, find the weight supported by each man.

Also find what would be the effect of placing each man one foot nearer to the centre of gravity of the pole.

4. Find the ratio of the power to the weight when there is equilibrium in a system of three moveable pulleys, each of which is supported by a separate string, and in which the free portions of the strings are vertical.

Also, if the weights of the pulleys, supposed to be equal, are taken into account, find the relation between the power, the weight, and the weight of a pulley.

5. A heavy uniform rod is supported by a string fastened to its ends, of double its own length, which passes over a smooth horizontal rail. Find the tension of the string first, when the rod is hanging at rest in a vertical position, and secondly, when the rod is at rest in a horizontal position.

6. Explain what is meant by saying that a point is moving in a straight line with uniform acceleration, and show how this acceleration is measured.

What is the measure of the acceleration of a body falling freely when eight feet and half a second are the units of length and time?

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7. A body is projected vertically upwards with the velocity of 256 feet per second; find the greatest height to which it rises and the time in which it will return to the point of projection.

Also find the times, during the ascent and descent, at which it passes the level of 768 feet above the point of projection.

8. Prove that the path of a projectile is a parabola, and that, if u is the horizontal component of the velocity of projection, the latus rectum of the parabola is equal to $\frac{u^2}{16}$.

9. The top of the spire of a church, standing on a level plane is 200 feet above the plane. From a position on the plane, at the distance of 400 feet from the vertical line through the top of the spire, a bullet is fired off so as to pass horizontally just over the top of the spire. Find the initial direction and the initial velocity of the bullet.

10. Find the direction and magnitude of the acceleration of a point moving uniformly in a circle.

A mass of 7 pounds, on a smooth horizontal plane, is fastened to one end of a string, 7 feet in length, and the other end is fastened to a fixed peg on the plane. The string is then straightened, and the particle is projected horizontally, at right angles to the string, with such a velocity as to describe its circular path in $5\frac{1}{2}$ seconds. Find the tension of the string in poundals, and also in pounds' weight.

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FURTHER EXAMINATION.

VI. PURE MATHEMATICS.

[*Full marks may be obtained for about* two-thirds *of this paper.*]

1. Show that the two perpendiculars, erected at the extremities of any chord of a circle, meet any diameter of the circle at two points equidistant from the centre; and contain a rectangle equal to the difference of the squares on the radius and on half the interval they intercept on the diameter.

2. Describe a circle passing through two given points and intercepting, on a given line, a segment of given length.

3. Define a homogeneous integral function of any number of variables.

Write down the most general function, of degree 4, in 3 variables.

4. If the increase in population be 8 per cent. every decade, the rate of increase being constant, find the population of a town of 100,000 inhabitants 5 years hence. Find also, employing a table of logarithms, the percentage of increase per annum.

5. Find the coefficient of xyz in the expansion of

$$\{(1-x-y-z)(1-x-y-z+4xyz)\}^{-\frac{1}{2}}$$

as a rational integral function of x, y, and z.

6. If the circle escribed to the side BC of a triangle ABC touch AB and AC produced in D and E respectively, prove that $AD = AE =$ half the sum of the sides of the triangle.

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The longest side of a triangular plot of ground is 100 yards, the perimeter is 250 yards and one angle is 40°. Determine the remaining angles.

7. If

$$r = p(1 + e\cos\phi)^{-1}$$
$$r' = p(1 + e\cos\phi')^{-1}$$
$$r'' = p(1 + e\cos\phi'')^{-1}$$

express

$$\frac{\sin(\phi'' - \phi')}{r} + \frac{\sin(\phi - \phi'')}{r'} + \frac{\sin(\phi' - \phi)}{r''}$$

in a form adapted to logarithmic computation and evaluate it where

$$\phi = 17^\circ\, 4', \quad \phi' = 22^\circ\, 27', \quad \phi'' = 38^\circ\, 19', \quad p = 21.$$

8. There is a rectangular plot of ground. Show how, by means of a cord, an ellipse may be inscribed so as to touch the sides at the middle points.

Prove the propositions on which the construction depends.

9. Give a geometrical construction for drawing tangents to an hyperbola from an external point.

10. Find the equation to a straight line passing through a fixed point (h, k) and making an angle of $\pi/3$ with the axis of x.

If the straight line rotate, in a counter clock-wise direction, about the fixed point through an angle of $1'$, show that the intercept on the axis of y is diminished by a length equal to $\frac{\pi}{2700} h$ approximately.

11. If the point (h, k) do not lie on the perimeter of the circle

$$(x - \alpha)^2 + (y - \beta)^2 - \rho^2 = 0$$

interpret the expression

$$(h - \alpha)^2 + (k - \beta)^2 - \rho^2$$

geometrically.

Transpose the above equation to polar coordinates and find the angle between the two tangents from the pole.

12. Find an equation to a parabola.

Prove, analytically, that, at every point of the curve, the diameter and focal radius make equal angles with the tangent.

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13. Given the axes of an ellipse, find expressions for

(i.) the distance between the foci.

(ii.) the distance between the directrices.

(iii.) the latus rectum.

(iv.) the product of the lengths of the perpendiculars from the foci on any tangent.

14. Explain carefully the nature of an asymptote to an hyperbola.

Find the equation to a curve, of this description, such that the smaller angle between the asymptotes is 45°, and the distance between the foci 10 units of length.

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VII. MECHANICS.

[*Full marks may be obtained for about* two-thirds *of this paper. Great importance is attached to accuracy.* N.B.—*g may be taken* = 32.]

1. Show how to find the resultant of a number of coplanar forces acting at a point.

Forces of magnitudes 3, 4, and 5, act at a point O in directions lying in one plane and making angles of 15°, 60°, and 135° respectively with a line OA in the same plane. Find to two places of decimals the magnitude of the resultant.

2. A small ring of weight W, which can move without friction on a circular wire fixed in a vertical plane, is in equilibrium at a point P on the lower half of the wire under the action of a force R in the direction of the tangent at P to the wire. If the pressure of the ring on the wire is equal to $\frac{1}{2}W$, find the magnitude and direction of the force R.

3. Define the centre of gravity of a rigid body. What assumptions with regard to the action of gravity are made for the purpose of the definition?

$ABCDE$ is a lamina of uniform thickness and density, and of such a shape that $BCDE$ is a square, and $AB = AE$. If the centre of gravity of the lamina is in BE, find the ratio of AB to BC.

4. Find the relation between the power and weight in the wheel and axle.

Show how to arrange three wheels and axles, having radii R and r respectively, so that $P/W = r^3/R^3$.

5. A body of weight 16 lbs. rests on a rough inclined plane inclined at an angle of 30° to the horizon. If a force of 2 lbs. acting up and parallel to the plane is just sufficient to prevent the body from slipping down, find the least force in the same direction which will balance the maximum resistance of the body to motion up the plane.

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6. State the proposition known as the "parallelogram of velocities."

Prove that if a point possesses two independent velocities represented by $\lambda . OA$ and $\mu . OB$, where OA and OB are two straight lines meeting at O, the resultant velocity will be represented by $(\lambda+\mu)OG$, where G is a point on AB such that $\lambda . AG = \mu . GB$.

7. A person rows with a velocity of 6 miles an hour across a river a quarter of a mile wide, which runs with a velocity of 4 miles an hour. The head of the boat makes a constant angle θ with the bank while he rows across, and he arrives at a point 36 yds. 2 ft. lower down the bank than the point opposite his starting point. Prove that $\tan\theta = \frac{1}{3}$.

8. Prove that if a point moves from rest with a constant acceleration f, the distance s passed over in a time t is given by $s = \frac{1}{2}ft^2$.

A body falls from the top of a tower, and after 2 seconds another body is projected downwards with a velocity of 192 feet per second. The two bodies reach the ground at the same time. Find the height of the tower.

9. A balloon when at a height of $2021\frac{1}{2}$ feet from the ground begins to fall with a uniform acceleration of $\frac{1}{16}g$. When the balloon is at a height of $500\frac{1}{2}$ feet from the ground, ballast to the amount of one-tenth the whole mass of the balloon is thrown downwards with a velocity, relative to the balloon, of 10 feet per second. Find the time the ballast will take to reach the ground.

10. A particle is projected from a point P with velocity v in a direction making an angle α with the horizon. Prove that the greatest height above P, to which the particle rises, is $\frac{v^2\sin^2\alpha}{2g}$.

A stone is thrown from a height of 4 feet so as just to pass horizontally over a wall which is 25 yards distant and 54 feet high. Find the velocity and direction of projection.

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MATHEMATICAL EXAMINATION PAPERS

FOR ADMISSION INTO

Royal Military Academy, Woolwich,

JUNE, 1895.

OBLIGATORY EXAMINATION.

I. EUCLID (BOOKS I.—IV. AND VI.).

[*Ordinary abbreviations may be employed, but the method of proof must be geometrical. Proofs other than Euclid's must not violate Euclid's sequence of propositions. Great importance will be attached to accuracy.*]

1. If ABC, DEF be two triangles which have the sides AB, AC equal to the sides DE, DF, each to each, and also the angle ABC equal to the angle DEF; then shall the angles ACB, DFE be either equal or supplementary.

2. Define *parallel straight lines, extreme and mean ratio*; and draw some simple figure of a *superficies* which is *not plane.*

If a straight line fall on two parallel straight lines, it makes the alternate angles equal to one another, and the exterior angle equal to the interior and opposite angle on the same side; and also the two interior angles on the same side together equal to two right angles.

3. In any right-angled triangle, the square which is described on the side subtending the right angle is equal to the squares described on the sides which contain the right angle.

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4. If a straight line be divided into two equal parts and also into two unequal parts, the rectangle contained by the unequal parts, together with the square on the line between the points of section, is equal to the square on half the line.

5. In obtuse-angled triangles, if a perpendicular be drawn from either of the acute angles to the opposite side produced, the square on the side subtending the obtuse angle is greater than the squares on the sides containing the obtuse angle, by twice the rectangle contained by the side on which, when produced, the perpendicular falls, and the straight line intercepted without the triangle, between the perpendicular and the obtuse angle.

ACB is a straight line, and on AC is described an equilateral triangle DAC; show that the square on DB is equal to the squares on AC and CB, together with the rectangle contained by AC and CB.

6. If two circles touch one another externally, the straight line which joins their centres shall pass through the point of contact.

7. In a circle the angle in a semicircle is a right angle; but the angle in a segment greater than a semicircle is less than a right angle; and the angle in a segment less than a semicircle is greater than a right angle.

$BEAC$ is a semicircle whose diameter is BC; D is any point on BC; AD is perpendicular to BDC; EB is equal to AD; and F is on DA produced so that DF is equal to AB; show that CE is equal to CF.

8. If from any point without a circle two straight lines be drawn, one of which cuts the circle and the other touches it; the rectangle contained by the whole line which cuts the circle, and the part of it without the circle, shall be equal to the square on the line which touches it.

9. Describe a circle about a given triangle.

10. What is Euclid's method for describing *about* a circle, a regular pentagon, hexagon, or quindecagon?

Show that this method would not apply to the describing about a circle of a triangle equiangular to a given triangle.

11. In a right-angled triangle, if a perpendicular be drawn from the right angle to the base, the triangles on each side of it are similar to the whole triangle, and to one another.

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12. Similar triangles are to one another in the duplicate ratio of their homologous sides.

$ADOB$ is the diameter of a circle whose centre is O; C is a point on the circumference such that CD is perpendicular to AB; and EC, EA are tangents to the circle; show that

triangle ECA : triangle OCB :: AD : DB.

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II. ARITHMETIC.

[N.B.—*The working as well as the answers must be shown.*]

1. Simplify $(2\frac{1}{4}$ of $5\frac{3}{7}) + (3\frac{1}{3}$ of $9\frac{1}{3}) - 10\frac{13}{35}$.

2. Divide 7·777 by 35·35.

3. In what time will the interest on £250, at $3\frac{1}{4}$ per cent. per annum, amount to £60. 18s. 9d.?

4. Reduce 3 cwt. 3 qrs. 21 lbs. to the decimal of 5 cwt.

5. Find the value of $5\frac{5}{12}$ of £13. 13*s*. 6*d*.

6. What is the rent of a farm of 246 acres, 3 roods, 24 poles, at £2. 5*s*. per acre?

7. Find the greatest common measure of 8775 and 12025.

8. If the water in a tank, 8 feet long and 7 feet wide, is 4 feet deep; and a cubic foot of water weighs 1000 ozs.; what is the weight in tons of the water in the cistern?

9. A certain field could be reaped by 7 men in a certain time, and 5 boys could do as much as 2 men. Find how many boys would be required, in addition to 30 men, for the reaping of a field of twice the size, in a third part of the time.

10. Show that the difference between any improper fraction and unity is always greater than the difference between unity and the reciprocal of the fraction.

[*You may take any improper fraction which you like to select, show that the proposition is true for that fraction; and then extend your reasoning to improper fractions generally.*]

11. The residue of an estate was left to be divided between three persons, *A*, *B*, *C*, in such proportion that *A*'s share was to be to *B*'s share as 4 : 5, and *B*'s share to *C*'s as 9 : 16; the residue realised £2,415. How much was each person entitled to?

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12. If you invest a sum of money in such ways that on one-third of it you gain 3 per cent., on one-fifth you lose 4 per cent., and on the remainder gain 6 per cent.; what average rate per cent. do you make on the whole sum invested?

13. Ten years ago a man was three times as old as his son, and five years hence he will be only twice as old. What are their ages respectively?

14. A farmer bought 6 oxen and 100 sheep for £336; of the sheep, 4 died, and the rest were sold at £2. 7*s*. 6*d*. each; and 2 of the oxen fetched £15 each. At what price must the remaining 4 oxen have been sold if the profit on the whole transaction amounted to 5 per cent.?

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III. ALGEBRA.

(Up to and including the Binomial Theorem, the theory and use of Logarithms.)

N.B.—*Great importance will be attached to accuracy.*

1. Find the squares of $x+y-2z+1$, and of $x+y-2z-1$. What is the value of the difference of these squares when $z=\frac{1}{2}(x+y)$?

2. Find the L.C.M. of

(1) $(x-1)^2(x^2+2)^3$, $(x-2)^2(x^2-1)$, (x^4-4);

(2) x^5-xy^4, x^9+x^3y, $x^6+y^6+x^2y^2(x^2+y^2)$.

3. Prove the identities

(1) $$\frac{x^3}{(x-y)(x-z)}+\frac{y^3}{(y-z)(y-x)}+\frac{z^3}{(z-x)(z-y)}=x+y+z;$$

(2) $$\frac{a^3-y^3}{a^4-y^4}-\frac{a-y}{a^2-y^2}-\frac{1}{2}\left\{\frac{a+y}{a^2+y^2}-\frac{1}{a+y}\right\}=0.$$

4. Solve the equations

(1) $x^2-17x+72=0$.

(2) $$\begin{cases}11x-13y=40\\10x-12y=35+\frac{9}{11}.\end{cases}$$

(3) $$\begin{cases}3x^2+4xy+5y^2=\frac{71}{16}\\5x^2+4xy+3y^2=\frac{9}{4}.\end{cases}$$

5. A and B have the same birthday. A's age is represented, on his birthday in the year 1895, by the two right-hand digits of the year in which B was born; also the product of A's age and the number represented by the two right-hand digits of the year in which *he* was born gives the year when B was 9 years old. Find their ages, A being older than B.

6. Find the sum of an infinite geometrical progression whose first term is a and ratio r.

To what restriction is r subject, and why?

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During any year the excess of births over deaths causes an increase of h per cent. of the population, and at the end of the year a fixed number A of people emigrate. Prove that at the end of n years a population P becomes

$$b^n P - \frac{b^n - 1}{b - 1} A,$$

where

$$b = 1 + \frac{h}{100}.$$

7. Investigate the number of different ways in which n men may stand in a row.

If two specified men are, neither of them, to be at either extremity of the row, show that the number of arrangements is

$$(n-2)(n-3) \times (n-2)!$$

8. Write down the $(s+1)^{\text{th}}$ term in the expansion of

$$(a+b)^n.$$

Find the sum of all the coefficients; and show, by using a table of logarithms that if $a+b=1$ and $n=10$, the first *term* is greater than the sum of all the remainder if a is greater than (about) ·933.

9. Eliminate a from the equations

$$x = \log_a b, \quad y = \log_a c.$$

Prove that

$$z = 1 + \frac{\log z}{1!} + \frac{(\log z)^2}{2!} + \frac{(\log z)^3}{3!} + \ldots.$$

Employ tables to find the square root of π to 5 places of decimals

10. If

$$y^6 + y^5 - 5y^4 - 4y^3 + 6y^2 + 3y - 1 = 0,$$

and

$$y = x + \frac{1}{x},$$

show that

$$\frac{x^{13} - 1}{x - 1} = 0.$$

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IV. PLANE TRIGONOMETRY AND MENSURATION.

(Including the Solution of Triangles.)

[N.B.—*Great importance will be attached to accuracy.*]

1. Explain the measurement of angles in *circular measure.*

If an angle contains A seconds and its circular measure is a, show that, approximately,

$$A'' = 206265 \times a.$$

Find from the tables the values of $\cos\left(\frac{5}{8}\right)$ and $\sec\left(\tan\frac{\pi}{6}\right)$ disregarding seconds.

2. Show how to construct the angle whose cotangent is $\frac{8}{15}$, and find—

(*a*) The sine of this angle;

(*b*) The cosine of its half.

3. If x and y are any two numbers, show that the equation

$$\sin\theta = \frac{2xy}{x^2+y^2}$$

always gives a real value of the angle.

With this value for $\sin\theta$, find the value of $\tan\frac{\theta}{2}$.

4. Prove that

$$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}.$$

Prove also that

$$\sin(A-B)+\sin(B-C)+\sin(C-A) = -4\sin\frac{A-B}{2}\sin\frac{B-C}{2}\sin\frac{C-A}{2},$$

assuming the necessary elementary formulæ.

5. Prove the formula

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}.$$

Show that $\cos^{-1}\frac{8}{17} - \tan^{-1}\frac{3}{5} = \tan^{-1}\frac{33}{64}.$

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6. Prove the formulæ

$$\cos 2A = 2\cos^2 A - 1,$$
$$\cos 3A = 4\cos^3 A - 3\cos A.$$

7. Show that the sines of the angles of a plane triangle are proportional to the lengths of the opposite sides, and deduce the relation

$$\cos\frac{A-B}{2} = \frac{a+b}{c}\sin\frac{C}{2}.$$

8. ABC is a plane triangle, and P a point in the side AB such that

$$\frac{AP}{BP} = \frac{m}{n}.$$

If the angle CPB is θ, show that

$$(m+n)\cot\theta = n\cot A - m\cot B.$$

If the angles ACP and BCP are α and β respectively, show also that

$$(m+n)\cot\theta = m\cot\alpha - n\cot\beta.$$

9. Given $b = 14$, $c = 13$, and $A = 67° 22' 48''$ in a triangle, find C by logarithmic calculation.

10. The radii of the circular faces of a frustum of a right cone are 12 and 8 feet, and the area of its curved surface is $20\pi\sqrt{241}$ square feet; find the thickness of the frustum.

Show that the vertical angle of the cone, of which this is a frustum, is $29° 51' 46''$.

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V. STATICS AND DYNAMICS.

[*Assume that* $\pi = \frac{22}{7}$, *and that* $g = 32$.]

1. If a number of forces, lying in one plane, act at a point, explain how their resultant may be found.

A, *B*, *C*, *D* are the angular points of a square taken in order, and forces represented in direction by the lines *AB*, *BD*, *DA* and *AC*, and in magnitude by the numbers 1, $2\sqrt{2}$, 3 and $\sqrt{2}$, act at a point; find their resultant graphically or otherwise.

2. Prove that two forces, whose lines of action intersect, have moments about a point in their plane, that are together equal to the moment of their resultant about the same point.

P is a fixed point on the circumference of a fixed circle; *PM* and *PN* are any two chords of the circle at right angles to one another; *Q* is any other fixed point whatever in the plane of the circle. Show that if *PM* and *PN* represent forces, the algebraic sum of their moments about *Q* is constant.

3. Define the centre of gravity of a body; and show that a body cannot possess more than one centre of gravity.

Prove that the centre of gravity of three particles of equal mass placed at the angular points of a triangular lamina of uniform thickness coincides with the centre of gravity of the lamina.

4. A bent lever consists of two uniform, heavy, straight rods, whose lengths are as 3 to 4; find the weight which must be attached to the end of the shorter rod in order that—the fulcrum being at the junction of the two rods—they may make equal angles with the horizon.

5. Find the relation between the Effort or "Power" and the Resistance in the case of the frictionless screw press, when motion is just about to take place.

The step—or distance between two threads—of a screw is 0·187 of an inch, the length of the arm (reckoned from the centre of the screw) on which the effort acts is 25 inches, and the effort is 11·9 lbs.; find the resistance when the screw is on the point of moving.

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6. Express, in feet per second, the difference between a velocity of 60 yards per hour and a velocity of $3\frac{1}{2}$ feet per minute.

What would be the average velocity of a body which went 30 yards at the first rate, and then 35 yards at the second rate?

7. Find the acceleration of a particle, which is in a state of uniformly varying motion (1) if the velocity increases from 5 feet per second to 8 feet per second, while the particle describes a space of 13 feet; (2) if the spaces described in the first and sixth seconds are 7 and 17 feet respectively.

8. A body slides from rest down a smooth inclined plane of length 192 feet and height 12 feet; find

(i.) the acceleration of the body while sliding;

(ii.) the velocity acquired in sliding from the top to the bottom of the plane;

(iii.) the time taken to get from the top to the bottom of the plane.

9. What is the meaning of uniform angular velocity? Find in radians per second the angular velocity of the minute hand of a clock, keeping correct time.

If T is the periodic time of a particle revolving uniformly in a circle of radius r, show that the acceleration of the particle directed towards the centre is $\frac{4\pi^2}{T^2}r$.

10. Show that a heavy body projected obliquely in vacuo will describe a parabola.

Find the direction of projection when the range in a horizontal plane is $4\sqrt{3}$ times the greatest height.

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FURTHER EXAMINATION.

VI. PURE MATHEMATICS.

[*Full marks may be obtained for about* two-thirds *of this paper.*]

1. Solve the equations

(i.) $(a-1)(x+1)(x+a^3) = x(a^4-1)$.

(ii.) $\left.\begin{array}{r} \dfrac{x}{y+a}+\dfrac{y}{x+a}=1 \\ x^2+y^2=b^2 \end{array}\right\}$.

2. The interest on a sum of p pounds for a certain time is i pounds, and the discount at the same rate of interest for the same time is d pounds. Show that

$$\frac{1}{p}=\frac{1}{d}-\frac{1}{i}.$$

3. The index n being a positive integer, show that

(i.) All the coefficients in the expansion of $(1+x)^n$ are integers;

(ii.) The coefficients of terms equidistant from the beginning and end of the same expansion are equal;

(iii.) The sum of the even coefficients equals the sum of the odd coefficients.

4. If $u_n = \sin^n\theta + \cos^n\theta$, prove that

$$\frac{u_3-u_5}{u_1}=\frac{u_5-u_7}{u_3}.$$

5. A person travelling uniformly at the rate of 45 miles per hour along a straight line of railway observes the altitude of the top of a distant steeple at intervals of ten seconds. If two consecutive observations be $11^\circ\ 41'$ and $18^\circ\ 35'$ respectively, find what must be the height of the steeple that the next observation may be identical with the first of the former two.

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6. AHK is an equilateral triangle, and $ABCD$ is a rhombus whose sides are equal to the sides of the triangle, and BC, CD pass through H, K respectively. Prove that the angle A of the rhombus is ten-ninths of a right angle.

7. Lines are drawn joining the angular points A, B, C of a triangle to any point O in its plane. Prove that the lines from the middle points of the sides BC, CA, AB respectively parallel to OA, OB, OC, meet in a point.

8. Show that the straight line joining the points (24, 16) and (−21, −14) passes through the origin, and determine the co-ordinates of the points of trisection of this line.

9. Explain how to find the length of the perpendicular from the point (h, k) on the line $x \cos \alpha + y \sin \alpha = p$.

Find the equations of the lines bisecting the angles between the straight lines

$$3x + 4y = 5, \qquad 12x - 5y = 41.$$

10. Prove that

$$x^2 + y^2 - 4x - 2y + 4 = 0$$

represents a circle, and find the length of its radius.

Show that the lines $x = 1$, $y = 2$ each touch the circle, and find the other co-ordinates of the points of contact.

11. Find the equation to a tangent to the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

in terms of the eccentric angle of the point of contact.

Prove that two tangents to an ellipse which are at right angles to each other intersect on a fixed circle concentric with the ellipse.

12. Find the equation to the tangent at any point of the rectangular hyperbola $xy = c^2$.

If $c \tan \theta$, $c \cot \theta$ be the co-ordinates of a point on the curve, show that the chord through the points θ and ϕ, where $\theta + \phi$ is constant, passes through a fixed point on the conjugate axis of the hyperbola.

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13. Given a chord of a parabola and the direction of the axis, show that the locus of the focus is a hyperbola whose foci are at the extremities of the given chord.

14. Through one of the vertices A, and the extremities P, P', of a double ordinate of an ellipse or hyperbola, a circle is drawn cutting the axis again in K. If G be the foot of the normal at P, prove that GK is of constant length.

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VII. MECHANICS.

[*Full marks may be obtained for about* two-thirds *of this paper. Great importance is attached to accuracy.* N.B.—*g may be taken* = 32.]

1. Find the magnitude of the resultant of two forces P, Q which act at a point, the angle between their directions being θ.

Find the resultant of two forces 3 lbs. and 5 lbs. which act at an angle of 60°; and show that its magnitude will be unaltered if either of the given forces be replaced by a force of 8 lbs. acting in the opposite direction.

2. State the necessary and sufficient conditions for the equilibrium of three parallel forces acting upon a rigid body.

A bookshelf supported at its extremities is just filled by two sets of books, the books of each set being placed together. One set consists of 14 volumes, each $1\frac{1}{2}$ inches thick and weighing $2\frac{1}{2}$ lbs.; the other consists of 12 volumes, each $1\frac{1}{4}$ inches thick and weighing 2 lbs. Find the pressures on the supports, the weight of the shelf being 8 lbs.

3. The weight and centre of gravity of a body, and also of a portion of the body, being known, show how to determine the centre of gravity of the remainder.

A figure is formed by taking away from a square the triangle whose angular points are the middle points of three of the sides. Find the position of its centre of gravity.

4. State the laws of Limiting Friction, and explain what is meant by the "Coefficient of Friction."

A uniform beam AB whose length is $12\frac{1}{2}$ feet rests with one extremity A on a rough horizontal plane AC and is kept from falling forwards by a cord BC, 20 feet long, whose extremity is attached to a fixed point C in the plane, directly behind the beam. If the beam be on the point of slipping when $AC = AB$, find the coefficient of friction.

5. Find the relation between the power and the weight in that system of pulleys in which all the strings are attached to the weight, the weights of the pulleys being equal.

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If there be one fixed, and two moveable, pulleys, find how far the weight can be practically raised, if it be initially 24 feet below the lower moveable pulley.

6. Explain how velocity is measured, and if u be the measure of a velocity when s feet and t seconds are the units of space and time, find its measure when the units are s' feet and t' seconds.

Compare the velocities of two particles, one of which describes 9 miles in two hours and the other 11 feet in 4 seconds.

7. Express the space passed over by a particle moving subject to uniform acceleration in terms of its initial and final velocities and the time occupied.

An engine-driver reduces the speed of a train (at a uniform rate) from 40 to 30 miles per hour in a quarter of a minute. Find the distance passed over in this time, and also the velocity of the train when half this distance has been described.

8. A ball of mass m impinges directly upon a ball at rest of the same size but of mass m'. Show that after impact the balls will move in the same direction or in opposite directions according as m is $>$ or $<$ em', e being the coefficient of elasticity.

9. A particle is projected from O with velocity u in a direction inclined to the horizon at an angle α. Prove that the equation of its path is

$$y = x \tan \alpha - \frac{gx^2}{2u^2}(1 + \tan^2\alpha),$$

the axes of x and y being the horizontal and vertical lines drawn in the plane of the motion through the point of projection.

Find the velocity and angle of projection of a particle which being thrown from the level of the ground just clears a wall 18 feet high at a distance of 36 feet from the point of projection, and strikes a wall parallel to the former and 60 feet beyond it, at a point 8 feet above the ground. The plane of projection is perpendicular to the walls.

10. A particle of weight W, attached by a string of length L to the vertex of a smooth cone whose axis is vertical and semi-vertical angle α describes a horizontal circle on the surface of the cone with uniform velocity v; find the tension of the string and the pressure on the surface.

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MATHEMATICAL EXAMINATION PAPERS

FOR ADMISSION INTO

Royal Military Academy, Woolwich,

NOVEMBER, 1895.

OBLIGATORY EXAMINATION.

I. EUCLID (BOOKS I.—IV. AND VI.).

[*Ordinary abbreviations may be employed, but the method of proof must be geometrical. Proofs other than Euclid's must not violate Euclid's sequence of propositions. Great importance will be attached to accuracy.*]

1. If two triangles ABC, DEF, have the sides AB, AC of the one respectively equal to the sides DE, DF of the other, and the angle BAC equal to the angle EDF, the triangles are equal in all respects.

2. Prove that triangles on equal bases and between the same parallels are equal to one another.

If two triangles have equal bases, but the height of one be double the height of the other, prove, by Euclid's methods, that one of the triangles is double the other.

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3. If the square on one side of a triangle be equal to the squares on the other two sides together, prove that these sides include a right angle.

4. If a straight line AB be divided internally at any point C, prove that the square on AB is greater than the squares on AC, CB together, by twice the rectangle contained by AC and CB.

If the base BC of a triangle ABC be bisected at D, prove that the squares on AB, AC are together equal to twice the squares on BD and DA together. Show that this includes some of Euclid's propositions, in Book II., as particular cases.

5. Show how to divide a given straight line into two parts, so that the square on one part may be equal to the rectangle contained by the whole line and the other part.

6. Define a tangent to a circle, and prove that it is at right angles to the diameter of the circle through its point of contact.

7. Show that any two opposite angles of a quadrilateral inscribed in a circle are together equal to two right angles.

Show that two opposite sides of a convex quadrilateral (*i.e.*, a quadrilateral without re-entrant angles) described about a circle are together equal to the other two sides together.

State a sufficient condition that it may be possible to inscribe a circle in a given convex quadrilateral, proving your result.

8. Through a point O interior to a circle two chords AOB, COD, are drawn; prove that the rectangle AO, OB is equal to the rectangle CO, OD.

9. Describe a circle interior to a given triangle to touch the sides of the triangle.

Show that four circles can be drawn to touch the sides of a triangle, three of them being exterior to the triangle.

10. Describe an isosceles triangle having each of the angles at the base double of the third angle.

11. If the vertical angle BAC of a triangle be bisected internally by a line cutting the base in D, prove that the ratio $BD : DC$ is equal to the ratio $BA : AC$.

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If the perpendicular from C, upon the bisector AD, meet AD in N, and O be the middle point of the base BC, prove that ON is half the difference of the sides AB, AC.

12. If in the triangles ABC, DEF, the angles BAC, EDF be equal, and the ratio $BA : ED$ be equal to the ratio $FD : CA$, prove that the triangles are equal.

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II. ARITHMETIC.

[N.B.—*The working as well as the answers must be shown.*]

1. Simplify

$$\frac{5\frac{1}{2} \text{ of } 3\frac{1}{3} \text{ of } 2\frac{1}{7} - 6\frac{1}{4} \text{ of } 3\frac{1}{5} \text{ of } 1\frac{1}{8}}{7\frac{1}{5} \text{ of } 3\frac{4}{7}}.$$

2. Divide 3·425 by ·002192.

3. What principal, if invested for 3 years at $2\frac{3}{4}$ per cent. per annum simple interest, will amount to £575. 10*s.* 7*d.*?

4. Find the value of

$2\frac{1}{7}$ of $3\frac{1}{5}$ of 4 lbs. 8 oz. 10 dwt. 12 grs. Troy.

5. Reduce £4. 18*s.* $10\frac{1}{2}$*d.* to the decimal of 5 guineas.

6. Find the cost of 276 tons 16 cwt. at £3. 18*s.* $11\frac{1}{2}$*d.* per ton.

7. Find the least common multiple of 385, 231, 165, 105.

8. A room, 21 ft. 4 in. long, 18 ft. 8 in. wide, and 15 ft. 6 in. high, is papered with paper 32 inches wide at one shilling a yard. What is the total cost?

9. If by selling a certain horse for £66 I should lose 28 per cent. of the cost at which I bought the animal, what is my loss?

10. Prove that the product of any two numbers which consist of three figures and four figures respectively must be a number consisting of not less than six nor more than seven figures.

11. A cubical box of external dimensions 17 inches each way would contain crushed ore of the value of £421. 17*s.* 6*d.* if it were made of material 1 inch thick; but by mistake it has been made of thicker material, and the difference in the value of the ore which it will hold is consequently £78. 17*s.* 6*d.*; what is the real thickness of the material?

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12. Three persons contribute sums of £250, £500, and £750 respectively, towards a venture, on the understanding that the profits shall be divided in such a way that the *rate* of interest which each receives shall be in proportion to the amount of his contribution. If the profits for a year amount to £245, how much will each of them receive?

13. A train, going at the rate of 72 miles an hour, overtakes another train 192 yards long, going in the same direction on a parallel line at the rate of 54 miles an hour, and completely passes it in three-fourths of a minute: find the time in which the trains would have completely passed one another, if they had been going in opposite directions, and the length of the faster train.

14. Supposing the quantity of land under barley in England to be this year half as much again as that under wheat, and the quantity under oats to be equal to the other two together; if the quantity under wheat next year be reduced by 25 per cent., and the quantity under barley increased by 5 per cent., the whole quantity remaining the same as before, by how much per cent. will the quantity under oats be increased?

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III. ALGEBRA.

(Up to and including the Binomial Theorem, and the theory and use of Logarithms.)

[N.B.—*Great importance will be attached to accuracy.*]

1. Divide $x^5+x^4-9x^3+33x^2-7x-49$ by x^2+3x-7.

2. Find the factors of

(i.) $x^2+16x+63$.

(ii.) $y^3-43a^2y+42a^3$.

(iii.) $x^7-14x^5+49x^3-36x$.

3. Find the highest common factor of $x^4-3x^3-5x^2-18x+4$ and $4x^4-18x^3-5x^2-3x+1$.

4. Prove that $a+b+c$ is a factor of $a^4+b^4+c^4-2b^2c^2-2c^2a^2-2a^2b^2$, and find the other factors.

5. Solve the equations

(i.) $x^2-11x+30=0$;

(ii.) $\left.\begin{aligned}4x+7y&=-1\\3x-y&=3\end{aligned}\right\}$;

(iii.) $\left.\begin{aligned}x^2+y^2&=5\\x^2-y^2&=\tfrac{3}{2}xy\end{aligned}\right\}$.

6. A man leaves half his property to his eldest son, three-quarters of the remainder to his second son, and four-fifths of what is then left to his third son. If there is still £100 not disposed of, find the amount of the whole property.

7. An express train is timed to run at a uniform speed from a point A to a point C. B is a point on the line such that AB is three-fifths of the whole distance AC. In running from A to B the train has an average speed $1\frac{1}{2}$ miles an hour below its normal amount, and is consequently two minutes late at B. The driver arrives punctually at C by running from B to C at an average speed $2\frac{3}{7}$ miles an hour above the normal amount. Find the normal speed, and the distance from A to C.

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8. If a, b, c, d are consecutive terms of a geometrical progression show that $a^2+(b+c)(b+d)=c^2+(a+c)(a+d)$.

The first term of a geometrical progression is 1, and the common ratio is $\frac{2}{3}$. Find the sum of the first six terms; and find also (by using logarithmic tables) the number n of terms which must be taken in order that the difference between the sum of the first n terms and the sum to infinity may be less than $\frac{1}{2\times 10^7}$.

9. Show that the least possible value of the expression $x^2+2px+q$, for real values of x, is $q-p^2$.

If α, β are the roots of the equation $x^2+2px+q=0$, find the value of $(\alpha-\beta)^2$.

10. There are six gentlemen and nine ladies at a lawn-tennis party, and two courts are available. In how many ways can two "double" sets (*i.e.*, two sets of four players each) be made up, each pair of players consisting of a lady and a gentleman; the particular courts and sides taken by the different pairs not being taken into account?

11. Write down the first six *coefficients* in the expansion of $(1-2x)^{-\frac{1}{2}}$.

Find the greatest *term* in the expansion of $(2+3x)^7$ when $x=\frac{4}{5}$.

12. Prove that $\log_a xy = \log_a x + \log_a y$.

Find, by using logarithmic tables, the value of

$$\frac{\sqrt[3]{100}}{\sqrt[2]{123}}$$

to six places of decimals.

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IV. PLANE TRIGONOMETRY AND MENSURATION.

(Including the Solution of Triangles.)

[N.B.—*Great importance will be attached to accuracy.*]

1. Write down the relation which exists between the measures of an angle in degrees and in radians.

What is the measure (i.) in degrees (ii.) in radians of an internal angle of a regular decagon?

2. Prove geometrically the formula

$$\sin 2A = 2 \sin A \cos A, \text{ when } A \text{ is} < 45°.$$

Express $1 - \sin^6 A - \cos^6 A$ in terms of $\sin 2A$.

3. Obtain the value of $\sin 54°$.

If $\sin 4\theta \cos \theta = \frac{1}{4} + \sin\frac{5\theta}{2} \cos\frac{5\theta}{2}$, find *one* value of θ.

4. Find an expression for all the angles which have a given tangent.

Find the general value of x that will satisfy the equation

$$\tan x - \sqrt{3} \cot x + 1 = \sqrt{3}.$$

5. Establish the identities

(i.) $(\operatorname{cosec} A - \sin A)(\sec A - \cos A) = (\tan A + \cot A)^{-1}$.

(ii.) $\dfrac{\tan \theta}{(1 + \tan^2\theta)^2} + \dfrac{\cot \theta}{(1 + \cot^2\theta)^2} = \frac{1}{2} \sin 2\theta$.

(iii.) $\sin^{-1}\frac{4}{5} + \sin^{-1}\frac{5}{13} = \sin^{-1}\frac{63}{65}$.

6. If $A + B + C = 180°$, prove that

$$\cos A + \cos B + \cos C - 1 = 4 \sin\frac{A}{2} \sin\frac{B}{2} \sin\frac{C}{2};$$

and if $A + B + C = 90°$, prove that

$$\sin 2A + \sin 2B + \sin 2C = 4 \cos A \cos B \cos C.$$

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7. Find an expression for the cosine of half of an angle of a triangle in terms of the sides.

Prove that in any triangle

$$\frac{a}{bc}+\frac{\cos A}{a}=\frac{b}{ca}+\frac{\cos B}{b}=\frac{c}{ab}+\frac{\cos C}{c}.$$

8. In any triangle if

$$\tan\phi=\frac{a-b}{a+b}\cot\tfrac{1}{2}C,$$

prove that (without regard to sign)

$$c=(a+b)\frac{\sin\frac{1}{2}C}{\cos\phi}.$$

9. The sides of a triangle are 237 and 158, and the contained angle is 58° 40′ 3·9″. Find the value of the base, without previously determining the other angles.

10. If r and R are the radii respectively of the circles inscribed in, and described about the triangle ABC, show that

$$r=4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}.$$

11. Three halfpennies are placed on a flat table in contact with one another, and with their centres forming an equilateral triangle.

Find the area of the space enclosed between them. If a fine string is wound tightly round them so that each of the free portions of the string is a tangent to two of the coins, what is the length of the string?

Diameter of a halfpenny = one inch. $\pi = 3·1416$, $\sqrt{3} = 1·73205$.

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V. STATICS AND DYNAMICS.

[*g may be taken* = 32.]

1. Show that forces may be represented by straight lines.

If a straight line AB represent a force of 1 lb.; construct the line which shall represent a force of $3\sqrt{2}$ lbs.

Three forces of 1, 1 and $\sqrt{2}$ lbs., acting at a point, are in equilibrium; find, graphically or otherwise, the angle between the greatest force and either of the other forces.

2. State the necessary and sufficient conditions that three parallel forces acting upon a rigid body may be in equilibrium.

A heavy uniform bar $ACDB$ rests in a horizontal position upon two fixed supports C, D, whose distance apart is 6 inches and equal to the length of the projecting part AC of the bar. If an upward force of 2 lbs. applied at A just lifts the bar off the support C, and a downward force of 8 lbs. at A justs lifts it off D, find the length and weight of the bar.

3. Three forces, which are not parallel, act in one plane upon a rigid body and keep it at rest; prove that their lines of action meet in a point.

A square lamina $ABCD$, whose weight is 4 lbs., can turn in a vertical plane about a hinge at A. Find the force which, acting along BC, will keep the square in a position with this side horizontal; find also the magnitude and direction of the hinge action at A.

4. Show how to determine the centre of gravity of a system of heavy particles lying in one plane.

Four equilateral triangular laminæ, each of side a, but of different weights, $3W$, $5W$, $5W$, $7W$, are placed with their sides in contact and with the heaviest triangle in the middle so as to form an equilateral triangle of side $2a$. Find the position of its centre of gravity.

5. Draw any system of pulleys with parallel strings by means of which a force may balance a weight seven times as great.

If the direction of the force be vertical, find through what distance its point of application must move in order to raise the weight through 5 feet.

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6. Enunciate and prove the proposition known as the "Parallelogram of Velocities."

From a ship sailing in a north-easterly direction at 15 miles an hour it is observed that a second ship is always south of it; supposing that this second ship is sailing eastwards, find the rate at which it is travelling.

7. A particle starts from rest subject to a given uniform acceleration a. Write down the formulæ connecting s, the space described, v, the velocity acquired, and t, the time during which the motion takes place.

What will these formulæ become if the particle *starts* with velocity v subject to a *retardation* a?

8. State the usual relation between the measure of the mass of a body and that of the force which produces in it an acceleration a. Express the unit of force in terms of the units of mass, space, and time.

At A, B masses of 2 and 3 lbs. are respectively placed, and each mass is acted upon by a force equal to the weight of the other, and in a direction from it towards the other. If they start from rest at the same moment, and meet at the end of 3 seconds, find the distance AB and the velocities of the masses at the moment of striking.

9. Two bodies of given masses are suspended at the ends of a string which passes round a smooth pulley without weight. Find the acceleration, and the tension of the string.

If the masses be 8 lbs. and 17 lbs., and the former be initially 4 feet below the latter, find after what interval the vertical distance between the bodies will be again 4 feet.

10. A particle is projected with a given velocity and in a given direction; find the range on a horizontal plane through the point of projection.

If the particle pass through a vertical plate at the highest point of its path and have its velocity diminished in consequence by one-third, find the corresponding diminution in the range.

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FURTHER EXAMINATION.

VI. PURE MATHEMATICS.

[*Full marks may be obtained for about* two-thirds *of this paper.*]

1. What is the present value of an annuity of £400 per annum for 20 years, beginning one year from the present date, allowing compound interest at 4 per cent. per annum?

2. Two cyclists start together to race from Cambridge to Saffron Walden and back. The faster rider, who maintains a speed of fifteen miles per hour, is $2\frac{3}{4}$ miles ahead of the other on reaching Walden. After covering six miles of the return journey, the tyre of the leader is punctured and occupies 25 minutes to repair. The other rider passes him 7 minutes before he is ready to start again. Find who wins the race, and at what pace the slower rider travels.

3. Show how to find the greatest term in the expansion of $(a+x)^n$ by the Binomial Theorem.

Employ the Binomial Theorem to find the values of $\cdot 9^7$ and $\cdot 99^4$ to five places of decimals.

4. If α, β are unequal values of θ satisfying the equation

$$a \tan \theta + b \sec \theta = 1,$$

find a and b in terms of α and β, and prove that

$$\sin \alpha + \cos \alpha + \sin \beta + \cos \beta = \frac{2b(1-a)}{1+a^2}.$$

5. ABC is a triangle, and D, E, F, are the middle points of BC, CA, AB respectively. If the lengths p, q, r of the lines AD, BE, CF are given, solve the triangle.

If the measurements of p, q, r are slightly incorrect, and the small errors be x/p, y/q, z/r, find the consequent error in the computed value of the angle A when $y = z = -2x$, and show that the calculated values of b and c are not affected by these errors.

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6. O is a fixed point on a circle, and PQ a chord of the circle such that the sum of the squares on OP and OQ is constant. Show that the middle point of PQ lies on a fixed straight line.

7. Prove that the perpendiculars from the angular points of a triangle on the opposite sides intersect in a point.

This point being called the orthocentre, let P, Q, R, S be the orthocentres of the triangles BCD, CDA, DAB, ABC, where A, B, C, D lie on one and the same circle; prove that the quadrilaterals $ABCD$, $PQRS$ are of the same size and shape.

8. What points are represented by the equations

$$x^2+y^2=13, \quad xy=6?$$

Show their relative positions on a figure, and demonstrate that the four points are the corners of a parallelogram.

9. Find the general equation to a straight line which passes through the point of intersection of the two given straight lines

$$ax+by=1, \quad lx+my=1.$$

Determine the equation of the straight line which passes through the origin and the point of intersection of the lines

$$5x-3y=11, \quad x+2y=10.$$

10. Find the equation of the circle which has its centre at the point (9, 4) and passes through the point (1, −2), and show that it also passes through the point (3, 12).

Find the equation to the straight line which is the polar of the origin with respect to this circle.

11. Show that the line

$$y=mx+\frac{a}{m}$$

always touches the parabola $y^2=4ax$.

Show that the middle points of chords parallel to this tangent lie on a straight line parallel to the axis of the curve and passing through the point of contact of the tangent.

12. Find the equation to the chord through two points on an ellipse whose eccentric angles are given.

If P be a point on an ellipse, A, A' the extremities of the major axis, show that the tangent at P intersects the diameter parallel to AP on the tangent at A'.

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13. Prove geometrically that the product of the perpendiculars from the foci of an ellipse upon any tangent is equal to the square of the semi-axis minor.

If one focus of an ellipse inscribed in a triangle be at the circumcentre, prove that the other is at the orthocentre of the triangle.

14. Show that tangents from an external point to a conic subtend equal angles at a focus.

If the tangent at a point P of a hyperbola meet an asymptote in Q, and SP, drawn from the focus S which lies within the branch of the curve on which P lies, meet the same asymptote in R, show that the triangle RQS is isosceles.

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VII. MECHANICS.

[*Full marks may be obtained for about* two-thirds *of this paper. Great importance is attached to accuracy.* N.B.—*g may be taken* = 32 *feet per second per second.*]

1. One end, B, of a light cord is fixed; the cord passes over a fixed peg, A, in the horizontal line through B, and the other end, C, of the cord hangs down vertically below A. If a mass of weight P is suspended from C, find the magnitude and direction of the pressure on the peg.

2. Give the definitions of the *coefficient of friction* and the *angle of friction* between two bodies. Describe also any method by which these magnitudes have been measured.

A mass of 190 lbs. is placed on a rough inclined plane the tangent of whose inclination to the horizon is $\frac{5}{12}$, the coefficient of friction being $\frac{1}{2}$; find the magnitude of the horizontal force which will just suffice to drag the body up.

What is the magnitude of the horizontal force which will just prevent the body from sliding down?

3. Define the moment of a force about an axis (or a *point*, explaining when this latter expression may be used).

A and B are two fixed points in a given plane, 10 inches apart; a force P acts through A, but has any direction whatever in the plane; if P has always a moment of 60 inch-pounds' weight about B, give a graphic representation of the various magnitudes and directions of P.

What is the least, and what the greatest, value of P?

4. A ladder, AB, 15 feet long, rests against the ground at A and against a rough vertical wall at B, the coefficients of friction at A and B being $\frac{2}{3}$ and $\frac{1}{2}$ respectively; the centre of gravity, G, is 6 feet from A; find the inclination to the horizon at which the ladder will be just about to slip.

If the ladder is placed at an inclination $\tan^{-1}\frac{1}{2}$, and a boy whose weight is $\frac{1}{6}$ of that of the ladder ascends it in this position, how far will he be able to go before the ladder begins to slip?

5. Which of the accelerations, 15 miles per hour per 2 minutes and 2 inches per second per second, is the greater?

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If a point moves from rest with the first of these, through how many feet will it move in 10 seconds?

6. If a particle slides down a rough inclined plane of inclination i, the coefficient of friction being μ, find its acceleration.

If $\tan i = \frac{3}{4}$, and $\mu = \frac{1}{2}$, in what time will the particle move from rest over 125 feet of the plane?

7. A mass of 8 ounces hangs from a spring balance in a balloon: what tension will be indicated by the balance,

(*a*) if the balloon is moving upwards with constant velocity;

(*b*) if it is moving upwards with an acceleration of 2 feet per second per second?

8. Enunciate the two principles on which the solution of the problem of the collision of two spheres depends.

Two spheres are approaching one another, their centres moving in one and the same straight line; their masses are 10 and 6 ounces, their respective velocities 40 and 60 feet per second, and their coefficient of restitution is $\frac{1}{5}$; find their velocities after collision.

If they are in contact for $\frac{1}{64}$ of a second, find, *in ounces' weight*, the magnitude of the mean pressure between them.

9. If a particle moves in a circle of radius r with a velocity v, prove that it has an inward normal acceleration equal to $\frac{v^2}{r}$.

If the mass of the particle is 2 ounces, the radius of the circle 8 inches, and the velocity at all points 16 feet per second, what is the magnitude of the resultant force acting on the particle in each position, and what is its precise direction?

Point out the erroneous conception involved in the term "centrifugal force."

10. State, in a general way, the kind of effect produced on the trajectory of a projectile by the resistance of the air.

If the resistance of the air can be neglected when a projectile is fired at an elevation $\tan^{-1}\frac{12}{5}$ with a velocity of 520 feet per second, when and where will the projectile strike an inclined plane passing through the point of projection, the inclination of this plane being $\tan^{-1}\frac{8}{15}$?

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MATHEMATICAL EXAMINATION PAPERS

FOR ADMISSION INTO

Royal Military Academy, Woolwich,

JUNE, 1896.

OBLIGATORY EXAMINATION.

I. EUCLID.

[*Ordinary abbreviations may be employed; but the method of proof must be geometrical. Proofs other than Euclid's must not violate Euclid's sequence of propositions. Great importance will be attached to accuracy.*]

1. ABC and DEF are two triangles. If the sides BC, CA, AB are equal to the sides EF, FD, DE respectively, prove that the angle BAC is equal to the angle EDF.

2. ABC is any triangle, D any point inside the triangle. Prove that BD and DC are *together* less than BA and AC, and that the angle BDC is greater than the angle BAC.

The angle BAC is bisected by a straight line meeting BC in E, and P is any point on this straight line within the triangle. Prove that

(i.) BA and AC are *respectively* greater than BE and EC;

(ii.) BA and AC are *respectively* greater than BP and PC.

3. Write down Euclid's axiom with regard to *parallel straight lines*.

OA, OB are two finite straight lines meeting at O. C and D are any two points in OA and OB respectively. Through C and D straight lines are drawn at right angles to OC and OD respectively. Prove that these straight lines, if produced indefinitely in both directions, must meet.

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4. C is a point in a finite straight line AB. Prove that the square on AB is equal to the squares on AC and CB, together with twice the rectangle contained by AC and CB.

Express this result algebraically.

5. ABC is an acute-angled triangle. AD is drawn perpendicular to BC. Prove that the squares on AB and BC are together equal to the square on AC and twice the rectangle contained by BC and BD.

If M is the middle point of BC, prove that the difference of the squares on AB and AC is equal to twice the rectangle contained by BC and MD.

6. If a chord and a diameter of a circle intersect at right angles, prove that the diameter bisects the chord.

AB is a diameter of a circle. PQ is a chord *not* at right angles to AB. AM and BN are drawn perpendicular to PQ. Prove that $PM = NQ$.

7. If two circles touch one another externally, prove that the straight line which joins their centres passes through the point of contact.

8. Show how to draw tangents to a circle from a given point outside it.

P and Q are points on a diameter of a circle produced, and are at equal distances from the centre. PR and QT are tangents from P and Q, the points R and T lying on opposite sides of PQ. Prove that $PRQT$ is a parallelogram.

9. Prove that angles in the same segment of a circle are equal.

AB and CD are two intersecting chords of a circle. If the arcs AD and BC are together equal to the arcs DB and CA, prove that AB and CD are at right angles to one another.

10. Show how to describe a triangle whose sides shall touch a given circle and whose angles shall be equal to the angles of a given triangle.

11. Show how to find a mean proportional between two given straight lines.

12. AB is a diameter of a circle, CD is a chord at right angles to it. If any chord AP drawn from A cuts CD in Q, prove that the rectangle contained by AP and AQ is constant for different positions of P.

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II. ARITHMETIC.

1. Simplify

$$\frac{17\frac{8}{7}}{9\frac{1}{11}} + 2\tfrac{3}{7} \times (9\tfrac{1}{2} - 2\tfrac{1}{17}) - \frac{5}{1 - \frac{3}{81}}.$$

2. Divide ·1154255 by ·00115.

3. In how many years will £1500 amount to £1781. 5*s*. 0*d*. at $2\frac{1}{2}$ per cent. per annum simple interest?

4. Find the value of ·30875 of a mile in furlongs, poles, and yards.

5. Express the sum of $7\frac{1}{2}$ guineas $3\frac{1}{2}$ crowns and $5\frac{1}{4}$ florins as a fraction of £53.

6. What is the rental of an estate of 645 acres 3 roods 25 poles at £2. 11*s*. 4*d*. per acre?

7. Find the least common multiple of 555, 1221, and 2035.

8. Find the cost of a carpet, for a room 24 feet long and $17\frac{1}{2}$ feet wide, at 3*s*. 3*d*. per square yard, if a margin 2 feet wide be left uncovered.

9. A watch is offered for sale for £5. 15*s*. 0*d*.; and, if that price is reduced by 5 per cent., the dealer who is selling it will still make $9\frac{1}{4}$ per cent. profit: how much did the watch cost him?

10. State the rule for determining the remainder in a division sum when it is worked by dividing by two factors of the divisor successively. instead of by their product.

As an example, reduce 107 lbs. to quarters by dividing by 4 and by 7; and explain why you do what you do to find how many pounds there are over.

11. A man goes bankrupt with £1160 assets. His liabilities are the present value of three loans, at simple interest, amounting in all to £1800; one of them obtained 8 years ago at 4 per cent.; another, double of the first, 4 years ago at 5 per cent.; and the third, equal to the sum of the other two, a year ago at 8 per cent. How much does each of the three creditors lose?

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12. A cistern 12 feet deep, of which the length is double the width, holds 21 tons of water: find the depth of another cistern which is $1\frac{3}{8}$ ft. shorter and 4 ft. wider than the other, and which holds the same quantity. One cub. ft. weighs 1000 ozs.

13. Taking the values of zinc and copper to be £17. 14*s.* 0*d.* and £73. 15*s.* 0*d.* per ton respectively, find how much of each metal there will be in a mass compounded of the two, which weighs 14 cwt., and is worth £23. 12*s.* 0*d.*

14. A ship, steaming towards a port in a fog, fires a signal gun which is answered from the port, as soon as heard, by another gun; the report of the latter reaches the ship $25\frac{1}{2}$ seconds after the first gun was fired; the ship fires again immediately, is answered as before, and this time the reply is heard in $24\frac{1}{2}$ seconds. If the sound travels 1100 feet per second, at what rate must the ship be steaming?

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III. ALGEBRA.

(Up to and including the Binomial Theorem, and the theory and use of Logarithms.)

[N.B.—*Great importance will be attached to accuracy.*]

1. Remove the brackets in

$$7a+6[b-5\{c+4(b-3(a+2c))\}],$$

and find its value when $a=2$, $b=3$, $c=1$.

2. Prove that $a\times b=b\times a$, where a and b may have any positive integral values.

Multiply together

$$x-y,\quad x+y,\quad x^2-xy+y^2,\quad x^2+xy+y^2.$$

3. Show that x^n-a^n is divisible by $x-a$ for all positive integral values of n.

Find the factors of

$$bc(b^2-c^2)+ca(c^2-a^2)+ab(a^2-b^2),$$

and

$$x^4+5x^3+5x^2-5x-6.$$

4. Prove that

$$x^4-15x^3+75x^2-145x+84 \text{ and } x^4-17x^3+101x^2-247x+210$$

have the same H.C.F. and the same L.C.M. as

$$x^4-13x^3+53x^2-83x+42 \text{ and } x^4-19x^3+131x^2-389x+420.$$

5. In an examination a candidate takes five compulsory and two optional papers, makes the same marks on each paper, and gets fifty marks too few to pass. On a second attempt he increases his marks on each paper in the ratio of 63 to 50, and omits one of the optional papers, thus securing a hundred marks more than before and passing. If all the papers have the same maximum, find what number of marks is required to pass.

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6. Solve the equations

$$\frac{3x-16}{x-3}+\frac{2x+3}{x-2}=\frac{5x-2}{x+1},$$

$$3x^4-16x^3+26x^2-16x+3=0,$$

and form the quadratic equation, the sum of whose roots is 3, and the sum of the cubes of whose roots is 7.

7. If $\frac{a}{b}=\frac{c}{d}$, prove that a, b, c, d are proportionals according to Euclid's definition.

Find what number must be subtracted from each of the numbers, 8, 10, 13, 17, that the remainders may be in proportion.

8. Of any number of terms in Arithmetical Progression, show that the sum of the rth term from the beginning and the rth term from the end is equal to the sum of the first and last terms.

If $\frac{a}{b+c}$, $\frac{b}{c+a}$, $\frac{c}{a+b}$ are in A.P., show that a^2, b^2, c^2 are also in A.P.; a, b, c being positive quantities.

9. Find the total number of combinations which may be formed out of n things.

A pack of cards consisting of four suits of eight cards each is dealt to three players, so that each has ten cards and the remaining cards lie on the table. The eight cards of each suit being numbered in order from 1 to 8, find in how many ways the cards can be dealt so that one player may hold ten of the highest twelve cards, and another ten of the lowest twelve.

10. Prove that the coefficient of x^p in the expansion of $(1-x)^{-(n+1)}$ by the Binomial Theorem is equal to the coefficient of x^n in the expansion of $(1-x)^{-(p+1)}$ both expansions being in ascending powers of x.

Hence (or otherwise) find the sum of the first twelve coefficients in the expansion of $(1-x)^{-5}$ in ascending powers of x by the Binomial Theorem.

11. What is meant by the *characteristic* of a logarithm? Show that it may be determined by inspection in the case of logarithms to the base 10.

Find the number of integers in the product of 2^{19} by 3^{17}, and find the first four integers of the product.

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12. At an election for eight representatives, each voter had seven votes. The candidates were six liberals, six conservatives, and two acceptable to both parties. All six liberals were defeated, and the average of their votes was 1596 less than the average of the votes of the six conservatives. Each of the non-party candidates obtained a number of votes equal to the sum of these two averages. Altogether 24,064 persons voted, there was no spoilt vote, and each voter gave his votes to seven different candidates. Assuming that the same proportion of each party voted for *both* the non-party candidates, but that otherwise the voting was on strict party lines, find (i.) what proportion of voters voted for both the non-party men, (ii.) how many of the voters were of each party, (iii.) the number of votes polled by the non-party candidates.

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IV. PLANE TRIGONOMETRY AND MENSURATION.

[N.B.—*Great importance will be attached to accuracy.*]

1. Prove that the circumferences of circles vary as the radii.

Assuming the ratio of the circumference to the radius to be $6\frac{3}{7}$, find the circular measure of (*i.e.* number of radians in) one of the angles of a regular figure of 44 sides.

2. Trace the changes in the cosine of an angle as the angle varies from $0°$ to $360°$.

Show that the value of $\cos(A+B)$ found from the usual formula is of the right sign; where A and B are each obtuse and less than $135°$.

3. Find an expression for all angles which have a given cosine.

Find the three values of x, when $x^3 - 3x = 2\cos 3A$.

4. Show that, if A is an acute angle,

$$2\sin\frac{A}{2} = \sqrt{1+\sin A} - \sqrt{1-\sin A};$$

and, assuming the value of $\sin 30°$, test by means of this formula and the Table of common logarithms the correctness of the value of $L\sin 15°$ as given in the Table of logarithmic sines.

5. Establish the identities:

(i.) $\cos^2 A + \cos^2 B + \cos^2(A+B) - 2\cos A\cos B\cos(A+B) = 1$;

(ii.) $\tan 50° - \tan 40° = 2\tan 10°$;

(iii.) $\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{13} = \tan^{-1}\frac{3}{8}$.

6. Prove that, in general, small changes of the tabular logarithmic sine of an angle are approximately proportional to the corresponding changes of the angle.

7. In a plane triangle, prove that

(i.) $\tan\frac{B-C}{2} = \frac{b-c}{b+c}\cot\frac{A}{2}$;

(ii.) $\frac{\cos(B-C)}{\sin B\sin C} + \frac{\cos(C-A)}{\sin C\sin A} + \frac{\cos(A-B)}{\sin A\sin B} = 4$.

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8. If, in a plane triangle, $a = 447001$, $c = 341387$, $C = 37° 22' 12''$, find the two values of B; and draw a figure showing the two triangles obtained.

9. A balloon is vertically over a point which lies in a direct line between two observers who are 2000 feet apart, and who note the angles of elevation of the balloon to be $35° 30'$ and $61° 20'$; find its height.

10. (i.) Express the area of a regular polygon in terms of the radius of the circumscribed circle.

(ii.) Show that the cube of the perimeter of a triangle, multiplied by $\tan\frac{A}{2}\tan\frac{B}{2}\tan\frac{C}{2}$, is equal to the product of the diameters of the three escribed circles.

11. (i.) The *internal* diameter of a hollow ball of uniform thickness is two inches, and the thickness is one-fifth of an inch; find the number of cubic inches of material in the ball. [$\pi = \frac{22}{7}$.]

(ii.) Draw a plan and find, in acres, the area of a field, from the following notes:

		Yards	
		to E	
		550	
to C	55	242	154 to D
to B	176	110	
		from A	

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V. STATICS AND DYNAMICS.

[*g may be taken* = 32 *feet per second per second.*]

1. Define force, and explain what are meant by the force of gravity and the tension of a string.

A weight of 10 lbs. hangs at the end of a uniform rope 12 feet long, whose other end is fixed. If the rope weighs 3 lbs., find its tension at the point 4 feet from the weight.

2. Enunciate the parallelogram of forces.

ABC is an equilateral triangle and D is the middle point of AB; prove that the resultant of forces represented in magnitude by AD, AC, is represented by $\sqrt{7}$ times AD.

3. Find the direction and magnitude of the resultant of two parallel forces acting in the same direction.

If a heavy body is partly supported by a string and partly by a smooth horizontal plane, prove that the string must be vertical.

4. Find the position of the centre of gravity of the area of a triangle.

If D is the middle point of one side BC of a triangle ABC, prove that the distance between the centres of gravity of the triangles ABD, ACD is one-third of BC.

5. Find what force, acting horizontally, will support a weight W resting on a smooth inclined plane, the base of the plane being three times its height; also find the pressure on the plane.

6. Find the ratio of the power to the weight in the system of pulleys in which the string round any pulley has one end fastened to a horizontal bar, from which the weight is suspended, and the other end to the next pulley, neglecting the weights of the pulleys and the bar.

If there are two movable pulleys, find the point on the bar from which the weight should be suspended.

7. Explain how uniform velocity and uniform acceleration are measured. Find the measure of the acceleration due to gravity when a yard and three seconds are the units of length and time.

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8. Enunciate and prove the parallelogram of velocities.

A ship is sailing due north at the rate of three miles an hour, and a passenger on board walks transversely across the deck at the rate of 4·4 feet per second; find his actual directions of motion as he walks one way or the other.

9. A stone is projected vertically upwards with a velocity of 64 feet per second; neglecting the resistance of the air, find the greatest height to which it rises, and the time of its ascent and descent. Also find the times at which its height will be 28 feet.

10. State Newton's Second Law of Motion, and show how it gives a method of measuring force.

A body whose mass is 4 lbs. is moving with a velocity of 64 feet per second. If it is brought to rest in 128 feet by applying a constant resistance, find the magnitude of the resistance.

11. If a particle move with uniform velocity v in a circle of radius r, prove that its acceleration is in the direction of the centre of the circle, and is equal to $\frac{v^2}{r}$.

A heavy particle on a smooth horizontal plane is attached by a string to a fixed point on the plane. If the string be straightened, and the particle be projected horizontally in a direction perpendicular to the string, compare the tension of the string with the weight of the particle.

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FURTHER EXAMINATION.

VI. PURE MATHEMATICS.

[*Full marks may be obtained for about* two-thirds *of this paper.*]

1. Find the length of a man's stride, if dividing the number of strides he makes per minute by 30 gives his speed in miles per hour.

Contrast the rate of striding of a runner who covers 100 yards in 9·8 seconds, with a stride of 7 feet, with the rate of pedalling of a bicyclist who, on a machine geared so as to be equivalent to one with a driving wheel of 80 inches diameter, rides 30 miles in one hour.

2. Indicate the method of solution of simultaneous linear equations.

Three trains, of lengths a, b, c (feet), are travelling with uniform velocities u, v, w (feet per second), in the same direction on equidistant parallel rails with their rear-most carriages in a straight line.

Show that the trains may all be cut by some straight line or other for a time

$$\frac{a+c}{2v-u-w} \text{ seconds,}$$

or, for a time,

$$\frac{2b}{u+w-2v} \text{ seconds,}$$

according as $2v \gtrless u+w$.

3. Prove that the number of permutations of n different things taken r at a time when each of the n things may be repeated is n^r.

In the decimal system of notation, how many numbers are there which consist of four digits? Prove that the sum of all such numbers is

$$49495500.$$

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4. O is the centre of a circle, and AOB a diameter. Circles are described upon AO and OB as diameters. Show that the circle described, so as to touch the large circle internally and the two smaller circles externally, has a diameter one-third that of the larger circle.

5. $ABCD$ is a square of which BD is a diagonal. Through A a straight line is drawn, meeting BC and BD, produced, if necessary, in H and K.

If p be the perpendicular distance of K from BC, show that the reciprocal of p is equal to the sum or the difference of the reciprocals of BH and BA, and distinguish the cases.

If the line through A be drawn at random, show that it is an even chance that p is half the harmonic mean of BH and BA.

6. Construct a quadratic equation, with rational coefficients, so that one root may be

$$2 \sin 18^\circ.$$

7. Indicate the operations necessary in order to determine the distance between two inaccessible points in the same plane as the positions where the necessary observations are made.

8. Find the polar equation of a straight line in the form

$$r = p \sec(\theta - \alpha).$$

Find the condition that this straight line may touch the circle

$$r^2 - 2lr\cos(\theta - \beta) + l^2 - a^2 = 0.$$

9. Find the general equation of a circle whose centre lies on the axis of x.

If the abscissae of the centres of two such non-intersecting circles be $+a$ and $-a'$, and their radii r and r', find the coordinates of points on the axis of x at which the circles subtend equal angles. Find the equation of the radical axis of the two circles.

10. Taking the principal axes of an ellipse as coordinate axes find its equation.

If the ellipse be rotated through an angle θ in the positive direction show that its equation becomes

$$\cos^2\theta\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)+\sin^2\theta\left(\frac{x^2}{b^2}+\frac{y^2}{a^2}\right)+\sin 2\theta \,.\, xy\left(\frac{1}{a^2}-\frac{1}{b^2}\right) = 1.$$

If the direction of rotation be reversed, how is this equation affected?

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11. Find the equation of the normal to an ellipse at a given point.

The equation of an ellipse is $\frac{x^2}{4}+y^2=1$. Find the coordinates of the intersection of normals at the points whose eccentric angles are 75° and 15°.

12. Find the equation of a hyperbola in rectangular coördinates.

Show that if a variable line form, with two fixed lines, a triangle of constant area, the locus of a point which divides the intercept made on the variable line in a given ratio, is a hyperbola.

13. In a parabola, prove that an isosceles triangle is formed by the focal distance of a point, the normal at the point and the axis.

Find the locus of the foot of the perpendicular from the focus on the normal.

(*This question is to be solved geometrically.*)

14. Prove that the feet of the perpendiculars from the foci on any tangent to an ellipse lie on a circle whose radius is equal to the semi-major axis.

In an ellipse, if a line be drawn through a focus making a constant angle with the tangent, prove that the locus of the point of intersection with the tangent is a circle. Find also its centre and radius.

(*This question is to be solved geometrically.*)

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VII. MECHANICS.

[*Full marks may be obtained for about* two-thirds *of this paper. Great importance is attached to accuracy.* N.B.—*g may be taken* = 32 *feet per second per second.*]

1. Explain the derivation of the triangle of forces from the parallelogram of forces.

Two forces are represented by the lines joining the middle points of opposite sides of a quadrilateral. Show that their resultant is represented in magnitude and direction by one of its diagonals.

2. Show that if any number of coplanar forces acting at a point are in equilibrium, the sum of their components resolved in any two directions at right angles to each other must each be zero.

A string 31 inches long passes through a small ring of 4 ounces' weight, and has its extremities fixed at two points 25 inches apart, and in the same horizontal line. Find the tension of the string in the position of equilibrium in ounces' weight correct to two places of decimals.

Find also the magnitude of the horizontal force which, applied to the ring, will cause it to rest at a point 7 inches from the nearer end of the string.

(*This question may be solved graphically or analytically.*)

3. Prove that a system of coplanar forces will be in equilibrium if the algebraical sum of the moments of the forces about any three points not in the same straight line vanishes in each case.

A triangular lamina ABC, whose sides BC, CA, AB are respectively 18, 24, and 30 inches in length, is placed in a vertical plane with BC, CA resting upon two fixed smooth pegs D, E, 20 inches apart and in the same horizontal line. If equal weights W, W be suspended from A and B, and the triangle be kept with AB horizontal by means of a string connecting C with the peg D; find the tension of the string and the pressures on the pegs, neglecting the weight of the lamina.

(*This question may be solved graphically or analytically.*)

4. Define the centre of mass (centre of gravity) of a heavy body, and show that if a heavy body be suspended from a fixed point its centre of mass must be vertically beneath the point.

In a lamina of any form a line AB of length c is taken, and it is observed that when the lamina is suspended from A, the line AB dips 30° below the horizon, and 45° when suspended from B. Find the distance of the centre of mass of the lamina from AB.

(*This question may be solved graphically or analytically.*)

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5. A uniform ladder rests with one end against a vertical wall and the other on the ground, inclined to the vertical at 45°. Compare the least horizontal forces which, applied to the foot of the ladder, will move it towards or from the foot of the wall, the coefficient of friction being the same for both ends of the ladder.

6. A particle projected with a velocity u moves subject to an acceleration a in the direction of motion. Find the space described in the nth unit of time.

A particle slides from rest down a smooth plane inclined at 30° to the horizon. Find the position of that length of 92 feet which is passed over by the particle in one second.

7. Two scale pans, each of 4 oz. mass, are connected by a light inextensible string, which passes over a smooth fixed pulley. If a mass of 2 oz. be placed in one pan, and a mass of 3 oz. in the other, find the tension of the string and the pressures of the masses on the scale pans.

8. Two spheres of elasticity e and masses m, m', moving with velocities u, u', impinge directly. Find their velocities after impact.

The centres of two equal billiard balls of radius a and elasticity $\frac{8}{9}$ move along the straight lines, whose equations are in rectangular coordinates

$$4y = 3(x - a),$$
$$5y = -12(x + a),$$

in such a manner that the line joining them is always parallel to the axis of x, and impinge at the origin. Find the equations of their lines of motion after impact.

9. Prove that the path of a projectile in vacuo is a parabola, and that the velocity at any point is that due to falling from the directrix.

10. A particle of mass m attached to a fixed point by a light string of length l, makes complete revolutions in a vertical plane under the action of gravity. If u be its velocity at the highest point, find its velocity in any other position, and also the tension of the string.

If the ratio of its maximum to its minimum velocity be $a : b$, show that the maximum tension of the string will be to the minimum in the ratio of $5a^2 - b^2$ to $5b^2 - a^2$.

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MATHEMATICAL EXAMINATION PAPERS

FOR ADMISSION INTO

Royal Military Academy, Woolwich,

NOVEMBER, 1896.

OBLIGATORY EXAMINATION.

I. EUCLID.

[*Ordinary abbreviations may be employed, but the method of proof must be geometrical. Proofs other than Euclid's must not violate Euclid's sequence of propositions. In the absence of special directions to Candidates, any of the propositions within the limits prescribed for examination may be used in the solution of problems and riders. Great importance will be attached to accuracy.*]

1. Give accurate definitions of the following geometrical terms:—superficies, rectilineal angle, circle, rhombus, postulate, axiom.

2. Draw a straight line perpendicular to a given straight line of unlimited length, from a given point without it.

Construct a square which shall have an extremity of one of its diagonals at a given point, and the extremities of the other diagonal on a given straight line.

3. Prove that if a side of any triangle be produced, the exterior angle is equal to the two interior and opposite angles, and that the three interior angles of every triangle are together equal to two right angles.

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From a vertex of an equilateral triangle a perpendicular is drawn to the opposite side, and upon this perpendicular another equilateral triangle is constructed; show that its sides are perpendicular to those of the original triangle.

4. Prove that in obtuse-angled triangles, if a perpendicular be drawn from either of the acute angles to the opposite side produced, the square on the side subtending the obtuse angle is greater than the squares on the sides containing the obtuse angle, by twice the rectangle contained by the side on which, when produced, the perpendicular falls, and the straight line intercepted without the triangle, between the perpendicular and the obtuse angle.

Prove that a triangle, the sides of which are three, four, and six inches in length, is obtuse-angled.

5. Construct a square which shall be equal to a given rectilineal figure.

6. Prove that the diameter is the greatest straight line in a circle, and that, of all others, that which is nearer to the centre is always greater than one more remote.

7. Prove that the opposite angles of any quadrilateral figure inscribed in a circle are together equal to two right angles.

8. On a given straight line describe a segment of a circle, containing an angle equal to a given rectilineal angle.

Through a given point draw a straight line which shall cut off from a given circle a segment containing an angle equal to a given rectilineal angle.

9. Construct an isosceles triangle, having each of the angles at the base double of the third angle.

Prove that the perpendicular drawn to one side of such a triangle, at its middle point, cuts the other side in extreme and mean ratio.

10. Prove that the sides about the equal angles of triangles which are equiangular to one another are proportionals.

The altitude of a certain triangle is equal to its base; show that if a rectangle be inscribed in it so as to have one side along the base and the extremities of the opposite side upon the sides of the triangle, then the three triangles by which the original triangle exceeds the rectangle are together equal to half the square on the diagonal of the rectangle.

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11. Prove that parallelograms which are equiangular to one another have to one another the ratio which is compounded of the ratios of their sides.

12. Prove that, in any right-angled triangle, any rectilineal figure described on the side subtending the right angle is equal to the similar and similarly described figures on the sides containing the right angle.

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II. ARITHMETIC.

1. Simplify

$$\frac{7\frac{1}{3}+3\frac{1}{7}}{11\frac{1}{3}-9\frac{1}{11}}\times\left(2\tfrac{3}{4}-\frac{3}{8\frac{1}{2}+6\frac{1}{6}}\right).$$

2. Divide 26·751 by ·000925.

3. Find the Least Common Multiple of 34, 42, 119, and 255; and obtain the sum of the fractions $\frac{3}{34}$, $\frac{1}{42}$, $\frac{10}{119}$, and $\frac{1}{255}$, by reducing them to a common denominator.

4. Find the value of $\frac{4}{15}$ of £1. 19*s*. 8½*d*.

5. Find the cost of 25 quarters 3 bushels of oats at 19*s*. 3½*d*. a quarter.

6. One side of a rectangular field is ·054 of a mile, and the adjacent side is ·1⅓ of a furlong. Find the length of each side in yards and feet, and express the area of the field as a fraction of an acre.

7. A sum of £377. 13*s*. 4*d*. lent at simple interest amounts in a year and a half to £396. 11*s*. 0*d*. What is the rate of interest?

8. A garden whose length is 67 ft. 9 in. has a path 4 ft. wide on the two sides and at one end: if it costs £4. 10*s*. 3¾*d*. to turf the remainder at 6*d*. a square yard, what is the width of the garden?

9. *A*, *B*, and *C* who are engaged on piece-work do amounts in the same time which bear to one another the proportion of 10, 9, and 14 respectively. What ought each to receive if the amount paid for the whole is £24. 15*s*.?

10. What is meant by an *odd* number and an *even* number?

Show that it is only necessary to look at the digit in the unit's place to ascertain if a number is odd or even.

Several numbers have to be added together. What is the condition that their sum should be odd?

11. A cistern is filled in 3½ hours by a pipe 3 sq. in. in cross section through which water flows at the rate of 6·4 miles an hour. What is the volume of the cistern?

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12. If coffee and chicory cost £8. 10*s.* and £2. 10*s.* per cwt. respectively, what is the proportion of coffee and chicory in a mixture of which 7 lbs. are worth 7*s.* 6*d.*?

13. A man buys goods and finds that the cost of carriage is 4 per cent. on the cost of the goods. He is compelled to sell at a loss of 5 per cent. on his total outlay; if however he had received £3. 5*s.* more than he did he would have gained 2½ per cent. What was the original cost of the goods?

14. *A* and *B* set out from the same place in the same direction and travel uniformly; after 9 days' travelling *A* finds he is 72 miles ahead of *B*: he then turns and travels back the distance *B* would travel in 9 days, he then turns again and overtakes *B* in 22½ days from the start. What is the rate of travelling of each?

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III. ALGEBRA.

(Up to and including the Binomial Theorem, and the theory and use of Logarithms.)

[N.B.—*Great importance will be attached to accuracy.*]

1. What are the factors of x^6+y^3?

Hence, or otherwise, show that

$$(1+6a+6a^2)^6+(3+12a+12a^2)^3 \text{ is divisible by } (2+6a+6a^2)^2;$$

and find the value of the quotient when $a=-\frac{1}{2}$.

2. Find three factors of

$$1-x^2-2x^3-2x^4-x^5+x^7;$$

and the H.C.F. of

$$6x^3-13x^2+19x-7 \text{ and } 9x^3-27x^2+41x-28.$$

3. Simplify

$$\frac{1}{y\left(\frac{x}{y}-\frac{y}{x}\right)}-\frac{1}{x\left(\frac{x}{y}+\frac{y}{x}\right)}+\frac{1}{\left(\frac{1}{x}+\frac{1}{y}\right)(x^2+y^2)};$$

and prove that $\sqrt{9\sqrt{6}+6\sqrt{12}}+\sqrt{9\sqrt{6}-6\sqrt{12}}$

is equal to $2\sqrt[4]{216}$.

4. Solve the equations:

$$\text{(i.)} \quad 6x=25(x^{\frac{1}{2}}-1);$$

$$\text{(ii.)} \quad \begin{cases} 4x^2-6xy+27y^2=5, \\ 8x^2-9y^2=1; \end{cases}$$

and show that, if x be real, $\dfrac{x^2-50x+625}{x-50}$ can have no positive value less than 100.

5. I sent cash to a grocer for a certain number of lbs. of sugar, at the rate of 7 lbs. for 1*s.* $1\frac{1}{2}$*d.* But before the order reached him the price of sugar had risen, and the money was sufficient only to buy a quantity less by $10\frac{1}{2}$ lbs. than that which I had intended; so I sent an additional 5*s.* $7\frac{1}{2}$*d.*, and received one-fifth as much again as I had at first ordered. Find the number of lbs. ordered at first, the rise in the price being less than a halfpenny a pound.

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6. Show that, in a scale of notation of which the radix is r, when the sum of the digits of any whole number is divided by $r-1$, there will be the same remainder as when the whole number is divided by $r-1$.

If A and B be numbers in the scale of 10, and if 5 and 7 be the remainders when the sums of the digits of A and B are respectively divided by 9; find the remainder when the sum of the digits of the product of A and B is divided by 9.

7. Find expressions for the sum and general term of an Arithmetical Progression.

In a series of right-angled triangles, one side has, in succession, the lengths 1×4 in., 2×6 in., 3×8 in., 4×10 in., etc., and the hypotenuse differs in length from this side by one inch; show that the lengths of the other side form an Arithmetical Progression.

8. Find the number of permutations of n dissimilar things taken r together.

Find the number of arrangements of three different letters which can be formed of the ten letters Q to Z; Q, when it occurs, being always followed by U.

9. Prove the Binomial Theorem when the index is a positive fraction, assuming its truth for a positive integer.

Calculate by logarithms the numerical value of the 6th term of the expansion, by the Binomial Theorem, of $(1-x)^{120}$, when $x = \cdot220793$; and show that it is nearly 10^5.

10. Write down the expansions for a^x, e^x and $\log_e(1+x)$, each in a series of ascending powers of x.

If $z = \cdot9999999999$, and $e = 2\cdot71828$; find, to four places of decimals, the value of $z+\frac{1}{2}z^2+\frac{1}{3}z^3+$ etc.

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IV. PLANE TRIGONOMETRY AND MENSURATION.

[N.B.—*Great importance will be attached to accuracy.*]

1. Show that the angle subtended by any arc of a circle at the centre, may fairly be measured by the ratio of the arc to the radius of the circle.

Why may it not be measured by the ratio of the chord of the arc to the radius of the circle?

2. Define the tangent of a positive angle less than 360°.

If the angle A be known to be positive and less than 360°, what possible values can it have when

(i.) $\tan A = \sqrt{3}$. (ii.) $\tan \frac{1}{2}A = -1$. (iii.) $\tan 2A = \tan A$?

3. Prove the expression for $\sin(A+B)$ in terms of $\sin A$, $\cos A$, $\sin B$, $\cos B$, showing that the result is true for all sizes of the angles A, B. Hence find all the trigonometrical ratios of 105°.

4. Prove that in any triangle the sides are proportional to the sines of the angles opposite to them; and that the cosine of any angle of the triangle is expressible, in terms of the sides, by the formula

$$2bc \cos A = b^2 + c^2 - a^2.$$

If the sides of a triangle be 4, 5, 6, find the cosines of the angles; and hence, with the table of logarithms, determine the smallest angle to the nearest minute.

5. In any triangle ABC prove that

$$\tan \tfrac{1}{2}(B-C) = \frac{b-c}{b+c} \cot \tfrac{1}{2}A.$$

If $b = 11$, $c = 3$, $A = 57°$, find B and C to the nearest second, using the table of logarithms.

6. Prove the relations

(i.) $\tan A + \tan B + \tan C = \tan A \tan B \tan C$ where $A + B + C = 180°$;

(ii.) $\sin^2 B \sin^2(C-A) + \sin^2 C \sin^2(A-B)$
$+ 2 \sin B \sin C \sin(C-A) \sin(A-B) = \sin^2 A \sin^2(B-C)$.

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7. What is the meaning of $\tan^{-1}x$?

Show that

(i.) $\cos^{-1}[xy-\sqrt{1-x^2-y^2+x^2y^2}] = \cos^{-1}x+\cos^{-1}y$,

(ii.) $\tan^{-1}\left(\dfrac{x+y+z-xyz}{1-yz-zx-xy}\right) = \tan^{-1}x+\tan^{-1}y+\tan^{-1}z$.

8. If D be the orthocentre of the triangle ABC, that is, the point of intersection of the perpendiculars drawn from A, B, C to the opposite sides, prove that the radius of the circle drawn through A, B, C is such that

$$R=\frac{BC}{2\sin A}=\frac{AD}{2\cos A}.$$

Prove also that, if O be the centre of the circle drawn through A, B, C,

$$OD^2 = R^2(1-8\cos A\cos B\cos C).$$

9. Prove that the area of any quadrilateral $ABCD$ is given by

$$\sqrt{(s-a)(s-b)(s-c)(s-d)-abcd\cos^2\tfrac{1}{2}(A+C)},$$

where $a = AB$, $b = BC$, $c = CD$, $d = DA$, $2s = a+b+c+d$, and A, C are the angles DAB, BCD respectively.

10. Prove that the area of a triangle is half the product of the base and height, and that the volume of a right circular cone is one third the product of the area of the base, and the height.

What is the height of a right circular cone when its volume is equal to that of a sphere of which the radius is equal to the radius of the base of the cone?

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V. STATICS AND DYNAMICS.

[*g may be taken* = 32 *feet per second per second.*]

1. When is a force said to be given? Explain why a force can be represented by a finite straight line.

2. Assuming the parallelogram of forces, enunciate and prove the triangle of forces.

Two forces are represented in magnitude and direction by AO, BO; show that their resultant is represented by twice CO, where C is the middle point of AB.

If the forces are represented by AO, OB; find the resultant.

3. Two parallel forces whose magnitudes are P and Q act at the points A, B, of a body, in the same direction, and the length AB is given. Determine the point at which the resultant intersects the straight line AB.

The extremities A, B of a uniform rod rest on two supports while a weight of 30 pounds is fastened at a point C such that AC is twice CB. Find the pressure at A, (1) when the rod is light, (2) when it has a weight of ten pounds.

4. Find the centre of gravity of a uniform lamina bounded by a parallelogram. Show that it coincides with that of four particles of equal mass placed at the angular points.

5. A weight W is placed on a smooth inclined plane whose inclination to the horizon is two-thirds of a right angle, and is supported by a force whose line of action makes with the horizon an angle half that of the plane. Find the magnitude of the force.

6. Find the relation of the power to the weight in a single moveable pulley, (1) when the straight portions of the string are parallel, and (2) when they are at right angles.

7. Explain how velocity is measured, (1) when uniform, and (2) when variable. Compare the velocities of two trains, when one moves at the rate of 45 miles per hour, and the other at the rate of 33 feet per second.

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8. A weight W is suspended from a point O by a light string. Find the tension when the point O ascends vertically, (1) with a uniform velocity, and (2) with a uniform acceleration equal to one-third that of gravity.

9. The diameter AB of a given circle is vertical, and A is the lowest point. Prove that the times of descent from rest, down all smooth chords ending at A are equal.

Prove also that the velocity of arrival at A is proportional to the length of the chord.

10. A body is projected vertically upwards with a velocity of 640 yards per minute; find the height at which the velocity is 16 feet per second.

The point A is vertically over B and the distance AB is 30 feet. Two particles are simultaneously projected, one downwards from A, and the other upwards from B, each with a velocity of 30 feet per second. Find the distance from the middle point of AB at which they meet.

11. Enunciate the laws of motion.

A particle moving in given direction AB, with a velocity of 32 feet per second, is acted on by a uniform transverse force whose acceleration is g and whose direction AC is perpendicular to AB. Find the position of th particle and its direction of motion at the end of one second.

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FURTHER EXAMINATION.

VI. PURE MATHEMATICS.

[*Only* 10 *questions to be answered.*]

1. Show that the problem of finding the number of terms of an arithmetical progression when the first term, the common difference and the sum are given, may have more than one solution. If the sum of n_1 terms be equal to the sum of n_2 terms, show that the number n_1+n_2 depends only upon the ratio of the first term to the common difference.

2. Prove the Binomial Theorem for a positive integral index.

Express in its simplest form the coefficient of x^n in the expansion of the fraction

$$\frac{1}{(1+x+x^2+x^3+x^4)^3}$$

in ascending powers of x.

3. Investigate the expansion of $\log(1+x)$ to base e in a series of ascending powers of x.

Obtain a formula which would be convenient for the calculation of $\log_e 2$.

Explain how to pass from $\log_e 2$ to $\log_{10} 2$.

4. Divide a circle into two segments so that the angle contained in one shall be seven times the angle contained in the other, proving the proposition on which the construction depends.

5. Prove that the three perpendiculars from the angles of a triangle to the opposite sides intersect in one point. If H be the intersection and D, E, F the feet of the perpendiculars from the angles A, B, C, respectively, show that the four points A, B, C, H are the centres of the four circles that can be described so as to touch the sides of the triangle DEF.

6. Investigate an expression for the cosine of an angle of a triangle in terms of the sides.

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If θ, ϕ be the greatest and least angles of a triangle, the sides of which are in Arithmetical Progression, show that

$$4(1-\cos\theta)(1-\cos\phi)=\cos\theta+\cos\phi.$$

7. In order to determine the height of a mountain a base was measured of 2750 feet. At either extremity of the base were taken the angles formed by the summit and the other extremity. These angles were 58° 28′ and 111° 53′. Also at the extremity from which the latter angle was taken the angular height of the mountain was 11° 19′.

Find the height of the mountain.

8. Show how to construct a regular pentagon. Choose the most convenient axes and write down the equations of its sides and of its inscribed circle.

9. Find the angle between the straight lines represented by

$$x^2+y^2=2xy\operatorname{cosec}a,$$

the axes being rectangular.

Find the area of the triangle formed by these lines and the line

$$lx+my+n=0.$$

10. Find the equation to the tangent to the parabola

$$y^2-4ax=0.$$

Trace the parabolas

$$x^2=8ay,\quad y^2=27ax$$

and show that they intersect at right angles and also at an angle $\tan^{-1}\frac{9}{13}$.

11. From P, a point on an ellipse whose centre is C, an ordinate PM is drawn parallel to the minor axis. On PM a point Q is taken such that $QC=PM$.

Find the locus of Q.

12. Establish an equation to a hyperbola. Prove that the eccentricity is the secant of half the angle between the asymptotes.

13. In an ellipse S, F are the foci and P, Q any two points on the curve. If tangents at P, Q intersect at T, show that the angles QTS, PTF are equal.

14. Prove that the rectangle contained by the intercepts made by any tangent to a hyperbola on its asymptotes is constant.

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VII. MECHANICS.

[*Full marks may be obtained for about* two-thirds *of this paper. Great importance is attached to accuracy.* N.B.—*g may be taken* = 32 *feet per second per second.*]

1. Three forces proportional to the sides of a triangle act at the middle points of these sides in directions perpendicular to them. Prove that the forces will be in equilibrium if they act all inwards or all outwards. Forces $k \cdot AB$, $k \cdot BC$, $k \cdot CD$, and $k \cdot AD$ act perpendicularly to the sides AB, BC, CD, DA of a quadrilateral at their middle points, the first three acting inwards and the last outwards. Find their resultant.

2. Explain how to find the resultant of a number of parallel forces applied at given points in a straight line.

A rectangular portmanteau 3 feet in length and 2 feet in height and weighing 56 lbs. is carried up a staircase by two men supporting it along the front and back edges of its bottom face. If this face be held at an inclination of 30° to the horizon, find, to two decimal places of a pound, what portions of the weight are supported by the two men, supposing the centre of mass of the portmanteau to be at its centre of figure.

3. A body is made up of a number of different portions whose centres of mass are all in one plane. If the positions of these centres of mass are given and also the weights of the several portions, find the position of the centre of mass of the whole body.

Prove that the centre of mass of any quadrilateral lamina may be found by the following construction:—Let E, F be the middle points of the diagonals of the quadrilateral, O their point of intersection. On OE, OF cut off $OM = \frac{2}{3}\ OE$ and $ON = \frac{2}{3}\ OF$ and complete the parallelogram $OMGN$ having OM, ON as adjacent sides. Then G will be the centre of gravity of the lamina.

(*This question may be solved either geometrically or analytically.*)

4. Find the mechanical advantage in the common screw-press working without friction.

The coupling between two railway carriages consists of a right-handed and left-handed screw at opposite ends of a long bolt, working in nuts attached to the two carriages, each screw having five threads to the inch.

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Find the force which when applied to the end of an arm 18 inches long will cause the carriages to be drawn together with a force of 1 ton, taking the ratio of the circumference to the diameter of a circle as $3\frac{1}{7}$.

5. Explain what is meant by the *angle of friction.*

A body is placed on a rough plane whose inclination to the horizon is 45° the angle of friction being 30°. Prove that the force which must be applied to the body in a horizontal direction in order to just prevent it from slipping down is the same as if the plane were made smooth and its inclination to the horizon were decreased to 15°.

(*This question may be solved either geometrically or analytically.*)

6. A slip-carriage is detached from a train and brought to rest with uniform retardation in two minutes during which time it travels two-thirds of a mile. With what velocity was the train travelling when the carriage was detached?

The weight of the whole train was 130 tons and that of the slip-carriage 10 tons. Before the carriage was detached the train was just kept going with uniform velocity, by the pull of the engine, the resistance due to friction amounting to 35 lbs. weight per ton. Supposing the engine to pull the train with the same force after the carriage is detached, find the acceleration.

7. In Atwood's machine, two weights of 5 lbs. and 7 lbs. are attached to opposite ends of a string which passes over a light smooth pulley, and a rider weighing 4 lbs. is placed over the smaller weight. Find the acceleration.

When the system has moved through one foot from rest, the rider is detached by coming in contact with a fixed ring. How far will the 5 lb. weight descend below the ring before coming to rest?

8. A mass of m lbs. is making n complete revolutions per second on a smooth horizontal table in a circle of radius r to whose centre it is connected by a string. Find the tension of the string expressed in pounds' weight.

In the Watt's governor of a steam-engine, two equal light rods OA, OB, 1 foot long, are freely hinged at one end O to a vertical axis, and to their other ends A, B are attached equal weights of 7 lbs. The whole system is revolving about the vertical axis so that the rods OA, OB make equal angles with the vertical on opposite sides of it. Taking $\pi = \frac{22}{7}$, find the number of revolutions per minute when the rods include an angle of 120°.

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9. A particle is projected *in vacuo* from a point O with a velocity whose horizontal and vertical components are 32 feet per second and 80 per second (upwards) respectively. Draw a careful figure exhibiting the positions of the particle after 1, 2, 3, 4, and 5 seconds respectively, and state the coordinates of the particle at these instants, referred to horizontal and vertical axes through O.

Find also the coordinates of the focus of the parabola described.

10. A sphere of mass M moving with velocity U overtakes and impinges directly on a sphere of mass m, moving with velocity u in the same straight line. If the elasticity of the spheres be perfect, find by how much the kinetic energy of the first sphere is decreased and that of the second increased.

Three equal spheres whose coefficient of restitution (*i.e.* elasticity) is $\frac{1}{2}$ are ranged at rest in a straight line. The first sphere is then projected in the same straight line with velocity U, so as to impinge directly on the second. Find the final velocities of the three spheres after the last collision has taken place between them.

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MATHEMATICAL EXAMINATION PAPERS

FOR ADMISSION INTO

Royal Military Academy, Woolwich,

JUNE, 1897.

OBLIGATORY EXAMINATION.

I. EUCLID.

[*Ordinary abbreviations may be employed, but the method of proof must be geometrical. Proofs other than Euclid's must not violate Euclid's sequence of propositions. In the absence of special directions to Candidates, any of the propositions within the limits prescribed for examination may be used in the solution of problems and riders. Great importance will be attached to accuracy.*]

1. Define carefully the following geometrical terms:—point, plane, plane angle, circle, parallelogram.

2. Prove that if two angles of a triangle are equal to one another the sides which are opposite to the equal angles are also equal to one another.

3. Prove that if the square described on one of the sides of a triangle is equal to the sum of the squares described on the other two sides of it, the angle contained by these two sides is a right angle.

4. Prove that if a straight line be divided into any two parts, the squares on the whole line, and on one of the parts, are together equal to twice the rectangle contained by the whole and that part, together with the square on the other part.

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5. Prove that in every triangle the square on the side subtending an acute angle is less than the sum of the squares on the sides containing that angle, by twice the rectangle contained by either of these sides, and the projection on it of the other side.

6. Prove that if two circles touch each other externally, the straight line which joins their centres passes through their point of contact.

7. Prove that if a straight line touch a circle the straight line drawn from the centre to the point of contact is perpendicular to the line touching the circle.

8. Prove that if from any point without a circle two straight lines be drawn, of which one cuts the circle and the other touches it; the rectangle contained by the whole line which cuts the circle, and the part of it without the circle, is equal to the square on the line which touches it.

9. Inscribe a circle in a given triangle.

10. Prove that in a right-angled triangle, if a perpendicular be drawn from the right angle to the base, the triangles on each side of it are similar to the whole triangle, and to one another.

11. Prove that similar triangles are to one another in the duplicate ratio of their homologous sides.

12. Prove that in equal circles angles at the centre have the same ratio which the arcs on which they stand have to one another.

13. If two sides of a triangle are produced and the exterior angles between the produced sides and the third side are equal to one another; prove, without using any proposition subsequent to Euclid I. 6 (question 2), that the triangle is isosceles.

14. Within a parallelogram $ABCD$ any point K is taken; a straight line drawn through K parallel to AB meets AD and BC at H and G respectively, and a straight line drawn through K parallel to AD meets AB and CD at E and F respectively. Prove that the squares on EH and FG together exceed or fall short of the sum of the squares on HF and EG by a rectangle independent of the position of K.

15. Construct a square of which one side shall lie along a given tangent to a given circle and of which the opposite side shall be a chord of the circle.

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16. The circle inscribed in the triangle ABC touches the side BC at D. Show that the circles inscribed in the triangles BAD and CAD touch each other.

17. Through the extremities of two fixed perpendicular radii of a circle a pair of parallel chords are drawn. Prove that the circle whose diameter is the line joining the middle points of the two parallel chords passes through a fixed point, and also prove that the centre of this circle lies upon a fixed circle.

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II. ARITHMETIC.

Arithmetical methods of solution are required, and all the working as well as the answers must be shown.

1. Find the cost of 49 cwt. 3 qrs. 14 lbs. at £13. 8*s*. 10*d*. per cwt.

2. Divide 5·4 by 0·00072 and the quotient by 1,470,568 to four places of decimals.

3. Find the greatest common measure of 27531 and 8740.

4. Add $\frac{1}{51}+\frac{2}{45}+\frac{3}{38}+\frac{4}{85}+\frac{5}{57}$.

5. Find (in its lowest terms) the difference between $\frac{1638}{2205}$ and $\frac{910}{1911}$.

6. The French metre is 3·281 English feet, the Rhenish foot is 12·356 English inches. Express 4·567 metres in terms of Rhenish feet to 3 places of decimals.

7. A sum of money put out at Simple Interest amounts to £688 when the rate is $2\frac{1}{2}$ per cent. and the time 3 years. What would the amount be if the rate were $3\frac{1}{4}$ per cent. and the time $2\frac{1}{8}$ years?

8. Show that if a square number ends in 6 the figure in the tens' place is odd; if it ends in any other number, the tens' figure is even.

9. If *B*'s wages are $\frac{7}{10}$ths of *A*'s, for how many days will a sum which pays *A*'s wages for 119 days pay the wages of both together?

10. If the manufacturer makes a profit of 25 per cent., the agent one of 8 per cent., and the shopkeeper one of 20 per cent., what is the cost to the manufacturer of an article which is sold in the shop for £32. 8s.

11. A man buys an article and sells it at a profit of 10 per cent. If he had bought it at 10 per cent. less, and sold it for 6*d*. less, he would have made a profit of 20 per cent. Find the cost price.

12. To do a piece of work, a contractor can employ two classes of workmen, whose wages are in the ratio of 17 to 13. If he employs the higher paid men (who work the faster) he pays £148. 15*s*. in wages, being £10. 10*s*. less than the lower paid men would cost him. Compare their rates of work.

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13. A milkman adulterates his milk as follows:—He takes a pint out of each gallon and replaces it with water. He then takes another pint out of the mixture and replaces that with water; and he repeats the operation a third time. If he then sells his adulterated milk at the price per gallon that the pure milk originally cost him, what is his gain per cent.?

14. A tradesman makes up his receipts monthly. His receipts for the first two months average £3 less than those of the first month; those of the first three months average £4 more than the average of the first two; of the first four £5 more than that of the first three; of the first five £2 less than that of the first four; and of the first six £7 more than that of the first five. If the average monthly receipts for the first six months are £611, what were the receipts for each month?

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III. ALGEBRA.

(Up to and including the Binomial Theorem, and the theory and use of Logarithms.)

[N.B.—*Great importance is attached to accuracy.*]

1. Show that

$$x^3y^2+y^3z^2+z^3x^2-x^2y^3-y^2z^3-z^2x^3$$

is divisible by each of the factors $x-y$, $y-z$, $z-x$, and find the expression of the second degree which is the other factor.

Find also the factors of

$$1-b-a^2+a^3b+a^2b^3-a^3b^3.$$

2. Prove that if

$$\begin{aligned} A &= bB+C, \\ B &= cC+D, \\ C &= dD+E, \\ D &= eE, \end{aligned}$$

all symbols representing rational integral algebraic functions of x, then E is the highest common factor of A and B, considered as functions of x.

Find the highest common factor of

$$12x^3-16x^2-73x+39 \text{ and } 18x^3+51x^2+20x-13,$$

and write down what the two expressions and their highest common factor become when $x=10$.

3. Find the product of

$$\sqrt{2+\sqrt{3}},\quad \sqrt{2+\sqrt{2+\sqrt{3}}},\quad \sqrt{2+\sqrt{2+\sqrt{2+\sqrt{3}}}},\quad \sqrt{2-\sqrt{2+\sqrt{2+\sqrt{3}}}},$$

the positive value of the root being taken in every case.

Show that, if $ax^3+3bx^2+3cx+d$ is divisible by $ax^2+2bx+c$ the former is a perfect cube, the latter a perfect square.

4. Solve the equations:

$$\text{(i.)}\quad \frac{1}{x}+\frac{1}{a}=\sqrt{\frac{1}{a^2}+\sqrt{\frac{1}{a^2x^2}+\frac{1}{x^4}}},$$

$$\text{(ii.)}\quad \begin{cases} 3x^2+6yz+3(y+z)+1=0, \\ 3y^2+6zx+3(z+x)+1=0, \\ 3z^2+6xy+3(x+y)+1=0. \end{cases}$$

In (ii.) account for the theoretical eight sets of solutions.

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5. Two vessels, holding respectively 80 pints and 20 pints, are filled, the first with wine, the second with water. Two other vessels, of equal capacity p pints, are filled from the first and second of these respectively and then emptied into the vessel from which they were not filled. Find the composition of the mixture in each after two such operations; and show that, if the two liquids have then the same composition, their composition was the same after a single operation, and will remain so after any number of operations of the same kind. Find the value of p necessary for this result.

6. Find two numbers, each a perfect square of four digits, such that their difference is equal to the cube of the difference of their square roots.

Show how every perfect cube may be expressed as the difference of two perfect squares, and express thus the cube of 13.

7. When is a series of quantities said to be in Harmonical Progression? Find the nth term of such a progression, having given the first and third terms.

If $a-b$, $a-c$, $a-d$ are in Harmonical Progression, prove that the following three series of quantities are equally so:

$$\begin{array}{rlll} \text{(i.)} & b-c, & b-d, & b-a, \\ \text{(ii.)} & c-d, & c-a, & c-b, \\ \text{(iii.)} & d-a, & d-b, & d-c. \end{array}$$

8. Find the total number of combinations which can be formed out of n different things.

My cousin belongs to a club of 30 members. In how many ways can I meet at least five of the members of the club, of whom my cousin may be one; and in how many ways can I meet five members or more, not including my cousin?

9. If $f(p)$ denote the series $1+p+\frac{p^2}{\underline{|2}}+\frac{p^3}{\underline{|3}}+\ldots$ to infinity, prove that $f(m)\times f(n)$ is identically equal to $f(m+n)$.

Hence deduce the series for the expansion of a^x in a series according to ascending powers of x.

10. Solve the equation

$$\left(\frac{295}{867}\right)^{3-x}=632\left(\frac{56}{99}\right)^{\frac{5x}{9}}$$

determining x correctly to 3 places of decimals.

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IV. PLANE TRIGONOMETRY AND MENSURATION.

N.B.—*Great importance is attached to accuracy.*

1. Define *circular measure*, and the *sine of an obtuse angle*; and explain the meaning of the symbol π.

The circular measure of each of the angles of a regular figure of 77 sides is calculated to be 3·06; determine the assumed value of π.

2. Explain the *convention of signs* with reference to directions in a plane.

If $225° < A < 270°$, and $0° < B < 45°$, establish the identity

$$\cos(A - B) = \cos A \cos B + \sin A \sin B;$$

and show that the sign of $\cos(A - B)$ as found from the identity is the proper sign according to the convention.

3. Express $\cos 3A$ in terms of $\cos A$; and hence find the value of $\sin 18°$.

4. Find a general formula for all the values of θ which satisfy the equation $$3 \operatorname{cosec} \theta - 4 \sin \theta = 4.$$

5. If $A + B + C = 2S$, prove that

$$\sin(B - C)\cos(S - A) + \sin(C - A)\cos(S - B) + \sin(A - B)\cos(S - C) = 0;$$

and if $\alpha = \beta + \gamma$, show that

$$\sin(\alpha + \beta + \gamma) + \sin(\alpha + \beta - \gamma) + \sin(\alpha - \beta + \gamma) = 4 \sin \alpha \cos \beta \cos \gamma.$$

6. Prove that, in a plane triangle

$$a^2 = b^2 + c^2 - 2bc \cos A;$$

and adapt this formula to the logarithmic calculation of a by means of a subsidiary angle. If two sides and the included angle of a triangle be respectively 521, 479, and 59° 49′ 28″, find the third side.

7. A and B are two points on one bank of a straight river, distant from one another 649 yards; C is on the other bank, and the angles CAB, CBA are respectively 48° 31′ 10″ and 75° 24′ 50″; find the width of the river.

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8. Prove that the diameter of a circle circumscribed to a triangle is equal to the product of one of the sides and the cosecant of the opposite angle.

The angle A of a triangle ABC, inscribed in a circle, is a radian; and the smaller arc BC is 20 ins.; find (by logarithms) the area of the smaller segment of the circle cut off by the chord BC. ($\pi = 3\frac{1}{7}$).

9. Prove that $\tan^{-1}\frac{5}{6} + \tan^{-1}\frac{7}{6} = \tan^{-1}72$; and find x, when

$$\cos^{-1}\frac{1}{\sqrt{1+x^2}} - \cos^{-1}\frac{x}{\sqrt{1+x^2}} = \sin^{-1}\frac{1+x}{1+x^2}.$$

10. (i.) The slant side of a cone is 25 feet, and the area of its curved surface is 550 square feet; find its solid content. ($\pi = 3\frac{1}{7}$).

(ii.) If a sphere and a cube have equal surfaces, compare their volumes.

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V. STATICS AND DYNAMICS.

[*The measure of the acceleration due to gravity may be taken to be* 32 *when the foot and second are the units of length and time.*]

1. Explain what is meant by the tension of a string and how it is measured.

A weight of 11 lbs. is suspended from a fixed point by a uniform string, which itself weighs 15 oz.; find the tension of the string at its extremities and at its points of trisection.

2. Find the resultant of two unequal forces acting on a rigid body in parallel but opposite directions.

AB is a heavy uniform rod, which is suspended from a fixed point by a string fastened to a point P of the rod, such that AP is one-third of AB. If the rod weighs 12 oz., find what weight, suspended by a string from the end A, will keep the rod horizontal. If the rod be then tilted round P into a different position in a vertical plane, will the equilibrium be disturbed?

3. Enunciate the proposition called the parallelogram of forces. Find the magnitude of the resultant of two forces equal to the weights of 51 lbs. and 68 lbs., acting at right angles to each other.

4. AB is a straight line and C is its middle point; find the centre of mass of three equal particles, placed at the points A, B, C.

If the particle at C be moved to a point D, such that ADB is an equilateral triangle, find the new position of the centre of mass.

5. Find the ratio of the Power to the Weight on a Wheel and Axle.

The radius of the wheel being three times that of the axle, find how far the weight will be lifted when the power is pulled down through the space of one foot.

6. Describe the common steelyard, and show that the distances between the graduations are proportional to the differences of the weights to which they belong.

7. Explain how velocity is measured. A railway train passes over 333 miles in 7 hours 24 minutes; find its average velocity in miles per hour and in feet per second.

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8. Explain how uniform acceleration is measured.

If a stone is let fall, in what time will it reach a depth of 144 feet, and what will be its velocity at that time? If its mass is 8 oz., express its energy at that time in foot-pounds.

9. A heavy body is placed on a smooth inclined plane and allowed to slide down. The inclination of the plane to the vertical being two-thirds of a right angle, find the time of descent down a length of 128 feet of the plane, and the velocity acquired.

10. Two weights are connected by an inextensible string, which passes over a smooth fixed pulley; if the weights are held so that the portions of string below the pulley are vertical, and are then let go, determine the motion of each weight and the tension of the string.

If one of the weights is treble the other, find in what time the greater weight will descend through 32 feet, assuming that the string is long enough for this motion to take place.

11. Find expressions for the vertical and horizontal displacements at any time of a projectile *in vacuo*.

If the velocity of projection, in a direction inclined to the vertical at two-thirds of a right angle, is 64 feet per second, find the greatest height attained by the projectile. Also find the range of the projectile on the horizontal plane through the point of projection.

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FURTHER EXAMINATION.

VI. PURE MATHEMATICS.

[*Full marks may be obtained for about* two-thirds *of this paper.*]

1. A, B, C are points in a straight line ABC; P, Q, R are the centres of squares described upon AC, AB, and BC respectively, Q and R being on the side of AC remote from P. Show that BP is equal to QR, and at right angles to it.

2. Given a point O, and three lengths OA, OB, OC, find the conditions under which the triangle ABC has a maximum area.

3. Write down the general term in the expansion of $(1-x)^n$.

Under what circumstances does the expansion (1) terminate, (2) converge, n being a real quantity?

Given that three consecutive coefficients are -20, 190, and -1140; find n.

4. A rectangle of perimeter b is inscribed in a circle of radius a. Find its sides and determine the limits of possibility of the problem.

5. Prove the formula

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots \textit{ad inf.}$$

Show that

$$\sum_{n=1}^{n=\infty} \frac{n^3}{n!} = 5e.$$

6. Prove that

$$2\cos 2x \operatorname{cosec} 3x = \operatorname{cosec} x - \operatorname{cosec} 3x.$$

Sum to n terms the series

$$\cos 2x \operatorname{cosec} 3x + \cos(2 \cdot 3 \cdot x) \operatorname{cosec} 3^2x + \cos(2 \cdot 3^2 \cdot x) \operatorname{cosec} 3^2x + \ldots.$$

7. From the top A of a cliff 597 feet high, the angle of elevation of a balloon B was observed to be $47° \; 20'$ and the angle of depression of its shadow S upon the sea was $61°$. A, B, and S being in the same vertical plane, and the altitude of the sun being $65° \; 31'$, find the height of the balloon above the sea level.

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8. In question 1, taking A as origin and AC as axis of x, find equations in rectangular co-ordinates to the lines BP, AR, CQ, QR, PQ, PR.

9. In a circle if any chord be drawn through a fixed point and tangents be drawn at its extremities, find the locus of their points of intersection.

Show that if one point lie on the polar of a second point, the second point will lie on the polar of the first point.

10. Find the polar equation of a parabola. Having given the lengths of two tangents to a parabola at right angles to one another, find the length of the latus rectum.

11. Express the length of the normal (terminated by the major axis) to an ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1.$$

in terms of the inclination of the normal to the major axis.

Show that the sum of the squares of the normals drawn at the extremities of conjugate semi-diameters and terminated by the major axis is

$$a^2(e^2-1)(e^2-2).$$

12. Show that the locus of the vertex of a triangle constructed on a given base, one of whose base angles is double the other, is an hyperbola whose transverse axis is two-thirds of the base.

13. If a circle touch a given circle and a given straight line the locus of its centre is a parabola.

14. If PM be the perpendicular upon the directrix from any point P on an ellipse, then MS, drawn through the adjacent focus S, meets the normal at P on the minor axis.

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VII. MECHANICS.

[*Full marks will be given for about* two-thirds *of this paper. Great importance is attached to accuracy. Gravitational acceleration may be taken* = 32 *feet per second per second, and* $\pi = \frac{22}{7}$.]

1. Investigate the conditions that three parallel forces, whose lines of action are intersected by a straight line ABC in A, B, C respectively, may be in equilibrium.

A solid cone of weight W, height h, and radius of base r, rests with its slant side in contact with a smooth horizontal plane. Find (i.) the least vertical force which, applied at the vertex, will move the cone, and (ii.) the force at the vertex which will keep it at rest with its axis horizontal.

2. Find the magnitude and direction of the resultant of a system of forces acting in one plane on a particle, the magnitudes and the directions of the forces being given.

Five forces P, Q, X, R, Y act at O, the centre of a regular pentagon $ABCDE$, in the directions OA, OB, OC, OD, OE respectively. If the system be in equilibrium and P, Q, R be given, find the magnitudes of X and Y. [N.B.—Sin $18° = \frac{1}{4}(\sqrt{5} - 1.)$]

3. The sides AB and CD of a quadrilateral lamina $ABCD$ are parallel and at a distance h apart. If $AB = a$, $CD = b$, find the distance of the centre of mass of the lamina from the straight line bisecting the opposite sides AD and BC.

A rectangle $ABCD$ is bisected by a straight line cutting AB in E and CD in F. Show that the locus of the centre of mass G of either part is a parabola, and that the tangent at G to the locus is parallel to the corresponding line EF.

4. A rigid body is in equilibrium under the action of four forces in one plane, whose directions are not all parallel, and whose lines of action are not concurrent. If the magnitude of one of the forces be given, and the lines of action of all, show how the magnitudes of the other three may be found by the *graphical method.*

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C and D are two small rough pegs, 1 foot apart in a straight line inclined to the horizon at an angle of 60°. A uniform rod $ACDB$ of 10 pounds weight and 3 feet in length is placed in this line, under C and above D, so that $AC = BD = 1$ foot. Find *graphically* the least weight which, suspended from the *upper* end B, will keep the rod in the above position, the angle of friction between the rod and the pegs being 45°.

5. Describe the common balance, and explain how its sensibility depends on the form of the beam.

6. Enunciate and explain the proposition known as the "Parallelogram of Velocities."

An insect crawls along the minute hand of a clock at the rate of 32 feet an hour. Find its velocity at the distance of one yard from the centre of the clock face.

7. A particle is projected up a rough inclined plane with velocity u. Find its position and velocity at the end of time t, the inclination of the plane to the horizon being i, and λ the angle of friction.

If the particle be projected with velocity due to falling from a point A above the point of projection, and through A a straight line be drawn at an angle λ to the horizon, show that this line will meet the inclined plane at the point where the particle comes to rest.

8. If a particle be moving in a vertical plane under the action of gravity, prove that its path must be a parabola whose axis is vertical, and that the velocity at any point varies as the normal (terminated by the axis).

Find the two directions in which a particle may be projected, with a velocity of 60 feet per second, so as to hit an elevated object whose horizontal and vertical distances from the point of projection are 48 and 20 feet respectively; and prove that the times of describing the two paths are as 4 to 13.

9. Two spheres of masses m_1 and m_2 impinge obliquely with velocities u_1 and u_2 in directions making angles α_1 and α_2 respectively with the line of impact; find their velocities and directions after impact, e being the coefficient of restitution.

Three equal billiard balls A, B, C are lying on a billiard table; A and B are in contact, and equidistant from C. The ball C, projected *very nearly* in the direction of the point of contact, strikes A first and B *immediately* afterwards. Show that after impact the velocities of A and B are in the ratio of 4 to $3 - e$.

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10. Define work, potential energy and kinetic energy; and explain what is meant by the principle of the conservation of energy.

Three strings are knotted together at C. One string passes round a smooth peg A and supports a weight P at its free extremity; a second passes round a smooth peg B in the same horizontal line as A, and supports a weight Q; the third hangs vertically and supports a weight R. Prove that the work done in carrying C from its position of equilibrium to the peg A is

$$2Qc\sin^2\tfrac{1}{2}\theta$$

where $AB = c$, and θ is the angle which BC makes with the horizon in the position of equilibrium.

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MATHEMATICAL EXAMINATION PAPERS

FOR ADMISSION INTO

Royal Military Academy, Woolwich,

NOVEMBER, 1897.

OBLIGATORY EXAMINATION.

I. EUCLID.

[*Ordinary abbreviations may be employed, but the method of proof must be geometrical. Proofs other than Euclid's must not violate Euclid's sequence of propositions. In the absence of special directions to candidates, any of the propositions within the limits prescribed for examination may be used in the solution of problems and riders. Great importance will be attached to accuracy.*]

1. Give definitions of *parallel straight lines*, *rectangle*, *sector of a circle*, *reciprocal figures*. State the enunciation of any proposition in the Sixth Book of Euclid in which *duplicate ratio* occurs, and give a numerical illustration.

2. Prove that if a straight line falls on two parallel straight lines it makes the alternate angles equal to one another.

3. To a given straight line, apply a parallelogram equal to a given triangle and having one of its angles equal to a given rectilineal angle.

4. Enunciate the two propositions which prove that, if P, Q, and R be three points in a straight line *in any order*, the sum of the squares on PR and QR differs from the square on PQ by twice a certain rectangle.

5. Divide a given straight line into two parts, so that the rectangle contained by the whole and one of the parts may be equal to the square on the other part.

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6. Prove that if two circles meet they cannot have the same centre.

7. Prove that if, from the point of contact of a tangent to a circle, a chord be drawn, the angles which the chord makes with the tangent are equal to the angles which are in the alternate segments of the circle.

8. Prove that if two straight lines cut one another within a circle, the rectangle contained by the segments of one of them is equal to the rectangle contained by the segments of the other.

9. Inscribe in a given circle an equilateral and equiangular hexagon.

10. Two obtuse-angled triangles have one acute angle of the one equal to an angle of the other, and the sides about the other acute angle in each proportionals; prove that the triangles are similar.

11. Prove that if four straight lines are proportionals, the rectangle contained by the extremes is equal to the rectangle contained by the means.

12. Prove that in equal circles, sectors have the same ratio which the arcs on which they stand have to one another.

13. If the vertical angle of an isosceles triangle is half the angle of an equilateral triangle, show that the base is greater than half of one of the equal sides.

14. $ABCD$ is a quadrilateral in which ABC is a right angle; and the square on AD with twice the rectangle $AB \cdot CD$ is equal to the sum of the squares on AB, BC, and CD; show that the angle BCD is also a right angle.

15. A and B are fixed points on a circle whose centre is C, and P is a moving point on the circle; if AP revolves round A at a given rate, find the relative rates at which BP and CP revolve respectively round B and C.

16. $ABCDEF$ is a regular hexagon inscribed in a circle whose centre is O, and P is any point on the smaller arc AB; show that the perpendicular from P on OC is equal to the sum of the perpendiculars from P on OA and OB.

17. If in a quadrilateral $ABCD$, CD touches the circle through A, B, and C; and the rectangle $AB \cdot AC$ is equal to the rectangle $BC \cdot CD$; show that AB touches the circle through A, C, and D.

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II. ARITHMETIC.

[*In order to obtain full marks arithmetical methods of solution are required, and all the working as well as the answers must be shown.*]

1. Find, in its lowest terms, the difference between $\frac{1001}{1365}$ and $\frac{935}{2295}$.

2. Find the product of $\cdot 052 \times 1\cdot 87 \times \cdot 0021$, and divide the result by the product of $3\cdot 5 \times 6\cdot 63 \times 1\cdot 10$.

3. Find the cost of carpeting a room 30 ft. long by 21 ft. wide with carpet 27 in. wide at 3*s.* 9*d.* per yard. How much would be saved by leaving 5 ft. at each end and $3\frac{1}{2}$ ft. at each side uncarpeted?

4. What vulgar fraction of half-a-crown is equal to £·012?

5. A man bought a horse for £37. 10*s.*, and sold him so as to gain 25 per cent. At what price was the horse sold?

6. The interest for one year on £3060 after income tax at 6*d.* per £ is deducted from it amounts to £99. 9*s.* Find the rate per cent. of interest.

7. Find the value of 100,000 rupees in francs—one rupee being 1*s.* $3\frac{1}{2}$*d.* and 25 francs 10 centimes being the equivalent of £1.

8. Show that the product of any three successive numbers is divisible by 6.

9. Upon a debt which is paid a year and a half before it is due, true discount of £11. 4*s.* is allowed, simple interest being reckoned at 4 per cent. per annum. What is the amount of the debt?

10. A cistern which is 9 ft. 4 in. long and 7 ft. 6 in. wide contains 6 tons 5 cwt. of water; if a cubic foot of water weigh 1000 ozs., what is the depth of water in the cistern?

11. If the surface of a cube be 491·306406 square inches, what is the length of its edge?

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12. *A* works 7 hours a day for 6 days, *B* 4 hours a day for 9 days, and *C* 5 hours a day for 7 days: if the wages paid to *A* and *B* together amount to £7. 16*s.*, what do *A*, *B*, and *C* each receive?

13. A man purchases a farm which on being let at £2 per acre pays him $3\frac{1}{8}$ per cent. on the investment; he purchases another which he lets for 2 guineas an acre, and it pays him $3\frac{3}{4}$ per cent. Compare the price which he paid per acre for the two farms.

14. A train overtakes two persons who are walking 2 miles and 4 miles an hour respectively, and completely passes them respectively in 9 seconds and 10 seconds: what is the length of the train and its speed in miles per hour?

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III. ALGEBRA.

(Up to and including the Binomial Theorem, and the theory and use of Logarithms.)

[N.B.—*Great importance is attached to accuracy.*]

1. Explain carefully the meaning of the terms *algebraic sum* and *algebraic difference*, giving illustrations.

If $a^3 - 3a^2 - 2a + 1 = x - y$ and $a^3 + 3a^2 - 2a - 1 = x + y$ find x and y; and write down the product of $a^3 - 3a^2 - 2a + 1$ and $a^3 + 3a^2 - 2a - 1$.

2. (i.) Prove that the difference of the expressions $ax^3 + bx^2 + cx + d$ and $am^3 + bm^2 + cm + d$ is divisible by $x - m$, and write down the quotient.

What are the factors of

(ii.) $x^3 + y^3 + z^3$, having given that $x + y + z = 0$?

(iii.) $a^3(b-c)^3 + b^3(c-a)^3 + c^3(a-b)^3$?

3. Resolve into two factors each of the expressions

(i.) $x^4 + 1$ and

(ii.) $2x^2 + 7x + 2$,

calculating numerical coefficients correct to three places of decimals.

4. Simplify

(i.) $[a^{\frac{1}{3}} b^{\frac{1}{2}} (a^{-\frac{1}{6}} b^{-\frac{1}{4}})^{-\frac{1}{2}}]^{-\frac{4}{3}}$,

(ii.) $\sqrt{2\left(\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} + \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}\right)}$.

5. Solve the equations

(i.) $8(x-1) - 2\sqrt{(x-1)} - 15 = 0$.

(ii.) $x/a = y/b = z/c$ and $lx^2 + my^2 + nz^2 = 1$, x, y, and z being the unknowns.

In equation (i.) discuss whether either of the solutions you obtain is irrelevant, and, if so, what equation they satisfy.

6. If $a : b = c : d$, prove that

$$(ma + nc) : (mb + nd) = (na + mc) : (nb + md).$$

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If $\frac{a}{b} > \frac{c}{d}$, prove that $\frac{ma+nc}{mb+nd}$ will be greater or less than $\frac{na+mc}{nb+md}$, according as m is greater or less than n, assuming all the letters to denote positive quantities.

7. If the first term of a geometric progression is a and the common ratio is r, prove the following results:

(i.) If $r < 1$ the sum to infinity is

$$\frac{a}{1-r}.$$

(ii.) If $r > 1$, and if the progression be continued *backwards* by placing terms *before* the term a, the sum to infinity of the terms so added is

$$-\frac{a}{1-r}.$$

8. Prove that the characteristic of the common (or Briggian) logarithm of a number N, when $N > 1$, is equal to the number of places that the first significant digit of N is to the left of the units' place, and state and prove a corresponding result when $N < 1$.

In the series $2+4+8+16+\ldots$ determine the number of digits in the 250th term, given $\log 2 = \cdot 30103$.

9. How many different arrangements may be made with the letters of the word *Llanfairpwllgwyngyll* taken all together?

How many of these arrangements begin and end with a *ll* and contain a third *ll* but not *four* consecutive *l*'s?

10. Prove the Binomial Theorem for a positive integral index, and extend it so as to embrace all *positive* values of the index.

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IV. PLANE TRIGONOMETRY AND MENSURATION.

[N.B.—*Great importance will be attached to accuracy.*]

1. Define the cosine of an angle, and trace its changes in sign as the angle increases from zero to four right angles.

The angle C of the triangle ABC is equal to a right angle, and the sides AC, BC are respectively 10 and 20 feet. A perpendicular CD is drawn from C on AB, find the lengths of CD, AD, and BD.

2. Investigate expressions for the sine and secant of an angle when the tangent is known. What signs should be given to the radicals when the angle is (1) 200 degrees, and (2) 300 degrees?

Prove also that

$$\sin^4\theta + \sin^2\theta = 2 - 3\cos^2\theta + \cos^4\theta.$$

3. Prove that $\dfrac{\sin 2\theta}{2\sin\theta} = \cos\theta$.

Deduce from this (or otherwise determine) expressions for $\sin\dfrac{\theta}{2}$ and $\cos\dfrac{\theta}{2}$ when $\sin\theta$ is given. Apply the results to find $\sin\dfrac{\theta}{2}$ and $\cos\dfrac{\theta}{2}$ when $\sin\theta = -\dfrac{1}{3}$, and the angle θ lies between 270 and 360 degrees.

4. Investigate a general expression for all the angles whose sine is the same as $\sin\alpha$. Write down an expression for all the angles whose cosine is -1.

Find all the angles such that the tangent is twice the sine.

5. If $B + C = 180°$, prove that

$$2(1 - \sin B \sin C) = \cos^2 B + \cos^2 C.$$

If $A + B + C = 180°$, prove that

$$1 - 2\sin B \sin C \cos A + \cos^2 A = \cos^2 B + \cos^2 C,$$

and if $A + B + C = 0$

$$1 + 2\sin B \sin C \cos A + \cos^2 A = \cos^2 B + \cos^2 C.$$

6. Prove that in any triangle the sines of the angles are as the opposite sides.

The angles A, B of a triangle are respectively 40° 30′ and 45° 45′, and the intervening side is 6 feet, find the smaller of the remaining sides.

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7. A tower standing on the edge of a cliff is viewed by a man lying down on the shore. The tower and the cliff immediately under the tower are found to subtend each an angle of 30°, while the distance of the eye from a point at the foot of that part of the cliff is 20 feet. Find the height of the tower.

8. The sides of a triangle being given, investigate an expression for the radius of the circumscribing circle.

Prove that in an equilateral triangle the radii of the inscribed, circumscribed, and escribed circles are as 1 : 2 : 3.

9. If $A+B-C=0$, prove that

$$\tan A \tan B \tan C = \tan C - \tan A - \tan B.$$

Find also the value of $\tan^{-1}x + \tan^{-1}\frac{1-x}{1+x}$.

10. Investigate an expression for the area of the curved surface of a right cone when the angle of the cone and the altitude are given.

It is required to divide a right cone into two portions by a plane parallel to the base, so that the surfaces of each portion (including the flat faces) may be equal. Find the ratio into which the slant sides are divided by the plane, and determine if the problem is always capable of solution.

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V. STATICS AND DYNAMICS.

[*The measure of the acceleration due to gravity may be taken to be* 32 *when the foot and second are the units of length and time.*]

1. Explain why it is that forces can be represented geometrically by straight lines, and enunciate the propositions called the triangle of forces and the polygon of forces. What are the converses of these propositions? Are they necessarily true?

2. Find the resultant of two parallel forces acting in the same direction.

Two workmen are carrying a heavy ladder on their shoulders; find the portion of the weight of the ladder supported by each workman, the centre of gravity of the ladder being at the distance of three feet from one workman and of six feet from the other.

3. If three forces, acting in one plane, balance each other, prove that their lines of action are either concurrent or parallel.

Three rods, lying on a horizontal plane, are jointed together so as to form a triangle ABC, and a tightened string connects a point P in AB with a point Q in AC; find the directions of the stresses at B and C, and determine, by a graphic construction (hand-sketch), the direction of the stress at A.

4. Having given the positions of the centres of gravity of a solid body and of a given part of it, find the position of the centre of gravity of the remainder.

ABC is a triangular area, and D, E, F are the middle points of BC, CA, AB: if G is the centre of gravity of the triangle ABC, and if H is the centre of gravity of the quadrilateral $BFEC$, prove that HG is one-ninth of AD.

5. Neglecting the weights of the pulleys, find the ratio of the power P to the weight W in the system of pulleys in which the string which passes round any pulley has one extremity fixed and the other attached to the pulley next above it, the portions not in contact with any pulley being all parallel.

If there are three pulleys, each of weight w, find the relation between P, W, and w.

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6. A man walking at the rate of three miles an hour on a straight road is passed at the same instant by two cyclists, riding in the direction in which he is walking, one at the rate of seven miles an hour and the other at the rate of eleven miles an hour; all going at the rates indicated for three minutes, what will then be the distances of the cyclists from the man walking?

7. If a body, starting with the velocity u, moves in a straight line with the uniform acceleration f, prove that the space s, passed over in the time t, is given by the equation

$$2s = 2ut + ft^2.$$

A body, moving with uniform acceleration in a straight line, is observed to pass over 24 feet in three seconds, and 42 feet in the three seconds following; find its acceleration.

8. A stone is projected vertically upwards with the velocity of 96 feet per second; neglecting the resistance of the air, find the height to which the stone would rise, and the time in which it would attain that height.

Also find the times at which the stone would be at the height of 128 feet above the point of projection.

9. Prove that the time of descent of a particle down any chord of a vertical circle beginning at the highest point of the circle is the same.

Find the line of quickest descent from a given point to a given straight line, the point and the line being in the same vertical plane.

10. A mass of 15 lbs. is held on a smooth horizontal table, at the distance of 4 feet from the edge of the table, and a fine string is attached to it, passing over the edge of the table and supporting a mass of one pound. If the 15 lb. mass be let go, find the velocity with which it will reach the edge of the table.

Describe the subsequent motion of the system.

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FURTHER EXAMINATION.

VI. PURE MATHEMATICS.

[*Full marks may be obtained for about* two-thirds *of this paper.*]

1. The side AB of a triangle ABC is produced to D. If BE be drawn bisecting the angle CBD so as to meet AC produced in E, show that the square on BE is equal to the difference of the rectangles $AE \,.\, EC$ and $AB \,.\, BC$.

2. Draw a straight line which shall touch two given circles.

Two circles being given in position and magnitude; draw a straight line cutting them so that the chords in each circle may be equal to a given line, not greater than the diameter of the smaller circle.

3. Explain why, when quadratic equations are formed to express the conditions of a problem, the resulting roots may exceed in number what appear to be required as answers to the problem.

What does a merchant pay for brandy, when selling it for £39, he gains as much per cent. as the brandy cost him in sovereigns? Give an interpretation to the negative root which presents itself.

4. A steamer is running due west at 20 knots (miles per hour) in latitude such that its clock must be put back at the rate of one minute every 10 miles.

Find the run between consecutive noons, and the increase of speed necessary to make the run 520 miles.

5. Establish the formula

$$\tfrac{1}{2}\log_e y = m + \tfrac{1}{3}m^3 + \ldots + \frac{1}{2n-1}m^{2n-1} + \ldots, \text{ where } m = \frac{y-1}{y+1}.$$

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Employ it to calculate $\log_e 10$ to four places of decimals, and thence calculate the multiplier which serves to convert the Napierian logarithms into ordinary or Briggian logarithms.

6. Show *a priori* that the radii of the circles circumscribed and inscribed to a triangle must be expressible as symmetric functions of the sides.

Determine such expressions.

7. A steamer running due west at 15 knots observes that a fixed point of an iceberg, which is drifting uniformly due south, bears 45° to the northward of the vessel's course. At intervals of five minutes the bearings are found to be 50° 5′ and 56° 35′ respectively.

Determine the rate of drifting of the iceberg.

8. Find the length of the perpendicular from the focus on a tangent to a parabola, $y^2 = 4ax$; the inclination of the tangent to the axis being 60°.

Find the locus of the vertex of a parabola having a given point as focus and touching a given right line.

9. Find the equation of a straight line perpendicular to a given straight line.

A parallel being drawn to the base of a triangle so as to cut the sides; find the locus of the intersection of perpendiculars to the sides through the points of cutting.

10. Write down the general equation of a circle.

Find the locus of a point from which lines being drawn to several given points the sum of their squares shall be of given magnitude.

11. Find the equation of an equilateral hyperbola, taking the asymptotes as axes.

A straight line PCM is drawn from a fixed point P to intersect a fixed straight line AB in C and is produced to a point M such that a perpendicular MD to the fixed line intercepts a segment CD of given magnitude. Show that M is on an equilateral hyperbola, and find its centre and asymptotes.

12. Show that a curve of the second degree can be described in general so as to satisfy five conditions.

If you are given (1) the centre, (2) a system of conjugate diameters as regards direction and point of intersection; to how many conditions are these data respectively equivalent?

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13. The exterior angle between any two intersecting tangents to a parabola is equal to the angle which either of them subtends at the focus.

Prove that if S be the focus and PQ the points in which two fixed tangents are cut by a third, the triangle SPQ will have its angles constant.

14. Prove that the straight line joining the foci of an ellipse subtends at the pole of any chord half the sum or difference of the angles which it subtends at the extremities of the chord. Distinguish between the cases.

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VII. MECHANICS.

[*Full marks will be given for about* two-thirds *of this paper. Great importance is attached to accuracy. Gravitational acceleration may be taken* = 32 *feet per second per second, and* $\pi = \frac{22}{7}$.]

1. Show how to find the magnitude and line of action of the resultant of a system of coplanar forces.

Draw a straight line AB, 3 inches long, and upon it as base construct a triangle ABC whose sides CA, CB are $2\frac{1}{2}$ and $1\frac{1}{10}$ inches respectively. If forces of 5 lbs. act from C to A and from C to B, and a force of 6 lbs. from B to A, find *graphically* the magnitude and line of action of the resultant of the three forces.

2. If M_1, M_2 be the algebraical sums of the moments of a system of coplanar forces about two points A, B, prove that the algebraical sum about a point C lying between A and B will be

$$\frac{M_1 . BC + M_2 . AC}{AB};$$

and *hence* show that, if the algebraical sums of the moments of such a system with respect to three points not in a straight line, be separately equal to zero, the sum will be zero for any other point in the same plane.

3. The lower end B of a uniform rod AB, of length $2a$, rests on a smooth horizontal plane BD, and the upper end A is supported by a string AC of length b, the extremity C being fixed at a point whose height above the plane is $2a$. Find (i.) the horizontal force which, applied at the lower end of the rod, will cause it to rest at a distance b from the point in the plane immediately below C, (ii.) the reaction of the plane at B, and (iii.) the tension of the string AC;—the weight of the rod being W.

4. A uniform wire is bent into the form of a triangle; find the position of its centre of gravity.

If the lengths of the sides be 3, 5, and 7 inches, and the triangle be suspended from the obtuse angle, prove that in the position of equilibrium the shortest side will be horizontal.

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5. State the laws of Limiting Friction, and explain what is meant by the "coefficient of friction" and the "angle of friction."

A lamina in the form of an equilateral triangle rests within a fixed rough hoop, the plane of each being vertical, and the radius of the hoop equal to the side of the triangle. Show that in the limiting position of equilibrium, the base of the triangle will be inclined to the horizon at an angle equal to twice the limiting angle of friction.

6. Prove that the work done in drawing a body up a rough inclined plane is equal to the work done in drawing the body along an equally rough horizontal plane through a distance equal to the length of the base of the inclined plane, together with the work done in lifting the body through the height of the plane.

A train whose mass is 96 tons commences the ascent of an incline of 1 in 80 at the rate of 45 miles an hour, the resistance being 7 lbs. per ton. If after travelling $\frac{1}{2}$ a mile the velocity be reduced to 30 miles per hour, find in foot tons the work done by the engine.

7. Find the tension of a light string which passes round a smooth fixed peg, and has unequal masses m_1, m_2 attached to its extremities.

Two bodies of equal mass hang from the ends of a light string which passes round a fixed peg, and when at rest are 9 feet and 16 feet above a fixed horizontal plane. A piece of the upper body breaks off and strikes the plane at the same moment that the lower body does. Find the ratio of the parts into which the upper body is divided.

8. Show that there are generally two directions in which a particle may be projected with given velocity from a given point O so as to pass through a point P.

If R be the difference between the two ranges on a horizontal plane through O, and H the difference between the two greatest heights attained by the particle in the two paths, prove that $R = 4H \tan\theta$, where θ is the angle OP makes with the horizon.

9. A smooth sphere impinges obliquely on a fixed plane; find the motion after impact, the elasticity being imperfect.

Two billiard balls P, Q of radius r stand upon a smooth table $ABDC$, their centres being at distances h_1, h_2 from the cushion AC and k_1, k_2 from the cushion AB. Find the direction in which P must be struck so that after impinging in succession upon AB, AC it may strike Q directly, taking e to be the coefficient of restitution of the indiarubber cushion.

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10. The edges of a groove cut in a smooth horizontal table are two concentric circles whose radii are 11 and 13 inches. A sphere of 1 lb. mass whose diameter is 3 inches moves in this groove with uniform velocity. Find the greatest number of revolutions it may make per second without leaving the inner edge, and if it make half this number, find the reactions at the points where it rests upon the edges.

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MATHEMATICAL EXAMINATION PAPERS

FOR ADMISSION INTO

Royal Military Academy, Woolwich,

JUNE, 1898.

OBLIGATORY EXAMINATION.

I. EUCLID.

[*Ordinary abbreviations may be employed, but the method of proof must be geometrical. Proofs other than Euclid's must not violate Euclid's sequence of propositions. In the absence of special directions to candidates, any of the propositions within the limits prescribed for examination may be used in the solution of problems and riders. Great importance will be attached to accuracy.*]

1. Enumerate (without proofs) the cases in which with certain data, two triangles are equal in every respect.

Show that the proof of Prop. 4, Book I., may require one of the triangles to be moved out of its plane.

2. Show that if a parallelogram and a triangle have the same base and altitude, the area of the first is double that of the second.

Hence prove that the area of a trapezium is half the sum of the parallel sides multiplied by the perpendicular distance between them.

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3. Show that in any right-angled triangle, the square which is described on the side subtending the right angle is equal to the sum of the squares described on the sides which contain the right angle.

4. ABC is a triangle, and P the foot of the perpendicular from C on AB. Show that the square of AC differs from the sum of the squares of AB and BC by twice the rectangle under AB and BP.

Hence, if M is the middle point of AB, find the length CM in terms of the sides of ABC.

5. Prove any proposition in Book II. which shows that the rectangle under the sum and difference of two right lines is equal to the difference between their squares.

Hence give a purely geometrical construction for a point which divides a given line AB into two parts, the difference of whose squares is a given area.

6. Prove that if a circle passes through the vertices of a quadrilateral, the sum of a pair of opposite angles in the quadrilateral is two right angles.

Prove that if the sum of a pair of opposite angles in a quadrilateral is two right angles, a circle can be described round it.

7. Show how to construct an angle which is one-fifth of two right angles. What use is made of this by Euclid in Book IV.?

8. Show how to inscribe in a given circle a regular polygon of 15 sides.

9. Find in the base of a given triangle two points which divide the base into segments having to each other the ratio of the two sides (giving the proof).

A and B are two given points, and CD a right line parallel to AB; find on CD a point whose distance from A is three times its distance from B.

What is the greatest distance between AB and CD that will allow of a real solution?

10. Define similar triangles.

In a right-angled triangle, if a perpendicular be drawn from the right angle to the base, the triangles on each side of it are similar to the whole triangle and to one another.

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11. Define a *mean proportional* between two magnitudes.

AB is a given right line; O is a given point on AB produced through B; find by a geometrical construction a point P on AB such that PO is a mean proportional between AO and BO.

12. In a given triangle ABC inscribe a rectangle, one of whose sides lies along AB and is twice the other side.

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II. ARITHMETIC.

[*In order to obtain full marks arithmetical methods of solution are required.*]

1. What is the least fraction which, added to the sum of $\frac{29}{12}$ and $\frac{15}{16}$, will make the result an integer?

2. It requires 1344 tiles, each 9 in. by $4\frac{1}{2}$ in., to cover a court-yard. What is its area?

3. *A* and *B* own a field in shares proportioned as 15 to 13. If *A*'s share is $\frac{3}{8}$ of an acre, what is the size of the field in square yards?

4. What decimal of 11 cwt. is 3 qrs. 21 lbs.? *Or*,

What decimal of 10 kilograms is 75 grams?

5. Find the product of ·0119 and 2·967. Divide this product by 21·93.

6. What amount of interest does £1020 yield per annum, if invested at $2\frac{3}{4}$ per cent.?

7. Find the least common multiple of 78, 84, and 90.

Three cyclists who are riding together have machines with wheels, respectively, 78, 84, and 90 inches in circumference; what is the least distance in yards that they must travel in order that their wheels shall be simultaneously in the same position as at starting?

8. Find the decimal equivalent to the fractional expression

$$\frac{(1-\frac{5}{13})\times 6\frac{1}{2}\times(\frac{1}{3}\div 2)}{\frac{4}{5}\times 11\frac{1}{4}}.$$

9. A box 4 feet long, 2 feet wide, and $1\frac{1}{2}$ feet deep (internal measurements) weighs 10 lbs. If it be filled with water weighing 1000 oz. for each cubic foot, what is the total weight of box and water?

10. *A*, *B*, and *C* go into partnership, *A* investing £4000, *B* £3000, and *C* £2000. If the profits for the first year are £1530, how must they be divided?

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11. A man who has invested £6480 in $2\frac{1}{2}$ per cent. Consols at 108, sells out at 112, and invests the proceeds in 6 per cent. preference shares (nominal value £100), thereby increasing his income by £90. At what price did he buy the shares? (Neglect brokerage.)

12. Write down all the prime numbers between 50 and 100. Can you give any reasons why not more than two of these prime numbers are consecutive odd numbers?

13. A Parliamentary grant is made at the rate of 5*s.* per head for all the children at elementary schools. If this grant is distributed at the rate of 5*s.* 9*d.* per child in town and 3*s.* 3*d.* per child in country schools, what percentage of the total number of children are in each class of school?

14. A batsman has a certain average of runs for 16 innings. In the 17th innings he makes a score of 85 runs, thereby increasing his average by 3. What is his average after the 17th innings? (There are no "not out" innings.)

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III. ALGEBRA.

(Up to and including the Binomial Theorem, and the theory and use of Logarithms.)

[N.B.—*Great importance is attached to accuracy.*]

1. Prove the truth of the following statements:

(i.) The sum of the cubes of three consecutive numbers is divisible by the sum of the numbers.

(ii.) The sum of the cubes of any three numbers diminished by three times the product of those numbers is divisible by the sum of the numbers.

2. Find the greatest common measure of $x^4 - 7x^3 + 15x^2 - 16x + 21$ and $x^4 + x^3 - 12x^2 + x - 3$.

Show that if a and b denote two numbers which have no common factor, then $a^2 + b^2$ and $a^2 - b^2$ have no common factor unless it be 2.

3. Express in their simplest forms

(i.) $$\frac{a}{(a-b)(a-c)} + \frac{b}{(b-a)(b-c)} + \frac{c}{(c-a)(c-b)};$$

(ii.) $$\frac{a^3(b-c) + b^3(c-a) + c^3(a-b)}{a^2(b-c) + b^2(c-a) + c^2(a-b)};$$

(iii.) $$\frac{a+b}{a^3 - b^3} + \frac{a-b}{a^3 + b^3} - \frac{2(a^2 - b^2)}{a^4 + a^2b^2 + b^4}.$$

4. Assuming that $x^m \times x^n = x^{m+n}$ for all values of m and n, find what meaning must be given to x^0 and $x^{\frac{p}{q}}$ where p and q denote positive integers.

Find the continued product of

$$\sqrt{\frac{a^{\frac{1}{3}}x^{\frac{1}{2}}}{b}}, \quad \left(\frac{b^{\frac{1}{2}}x^{-\frac{1}{3}}}{y}\right)^{\frac{1}{2}}, \quad \left(\frac{ax^{\frac{1}{3}}y^{\frac{1}{2}}}{b^{-\frac{1}{3}}}\right)^{\frac{3}{2}} \text{ and } \frac{1}{a(by)}.$$

5. Show that a quadratic equation cannot have more than two roots.

The difference of the roots of a quadratic equation is d, and the quotient obtained by dividing the greater root by the smaller is q. Find the quadratic equation.

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6. Solve the equations

(i.) $\sqrt{ax+b}+\sqrt{ax+c}=\sqrt{2(b+c)}$;

(ii.) $3x-11y+7=5x+2y-8=11x-12$;

(iii.) $\dfrac{3x-1}{2x+7}+\dfrac{2x+7}{3x-1}=4\frac{13}{18}$.

7. If $\dfrac{bx+cy}{b+c-a}=\dfrac{cx+ay}{c+a-b}=\dfrac{ax+by}{a+b-c}$, and no numerator or denominator vanishes, show that either $x=y$ or $a=b=c$.

8. In how many different ways can the letters of the word *commission* be arranged when taken all together? In how many of these will the fourth and fifth places be occupied by the same letter?

I have two five pound notes, two sovereigns, two half-sovereigns, two florins, a shilling, and a sixpence; in how many ways can I give a donation?

9. By the aid of the binomial theorem, or otherwise, find the sum of the series

$$1+n+\frac{n(n-1)}{2!}+\frac{n(n-1)(n-2)}{3!}+\ldots,$$

n being an integer. (*N.B.*—$2!$ denotes *factorial* 2.)

Find the term in the expansion of $\left(x+\dfrac{1}{x}\right)^{2n}$ which is independent of x.

10. Write down the expansions in powers of x of a^x and $\log_e(1+x)$ as far as x^5, and state any limitation which may be necessary for the validity of either expansion.

Find the coefficient of x^n in the expansion of $\log_e(1+x+x^2+x^3)$ in powers of x, distinguishing the various cases which may arise.

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IV. PLANE TRIGONOMETRY AND MENSURATION.

[N.B.—*Great importance will be attached to accuracy.*]

1. Show how to connect the measure of an angle in degrees with the measure of the same angle in radians.

State (1) the numbers of degrees, and (2) the numbers of radians, in the angles described by the hands of a clock between 12 noon and 1.25 p.m. on the same day.

2. Define the sine of an angle, and prove that its value cannot exceed unity.

Write down the measures in degrees of the angles not exceeding two right angles whose sines are equal to 0, $\frac{1}{2}$, 1.

3. Obtain a formula for all the angles which have the same tangent as a given angle α.

If
$$\left.\begin{aligned} r\cos\theta &= a, \\ r\sin\theta &= b, \end{aligned}\right\}$$
and r, a, b are all positive, find a formula giving all the possible values of θ.

4. Prove geometrically the formula
$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$
for the case where the angles A, B and $A+B$ are acute.

Prove that, if $\sin x = n\sin(x+2\alpha)$,

then
$$\tan(x+\alpha) = \frac{1+n}{1-n}\tan\alpha.$$

5. Prove that, if A, B, C are the angles of a triangle,

(i.) $\tan A + \tan B + \tan C = \tan A\tan B\tan C$;

(ii.) $\cot A + \cot B + \cot C = \operatorname{cosec} A\operatorname{cosec} B\operatorname{cosec} C + \cot A\cot B\cot C$.

6. Prove that the limit of $\frac{\sin\theta}{\theta}$ when θ is indefinitely diminished is unity.

Find approximately in seconds, the greatest angle than can be subtended by a yard measure at a distance of one mile.

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7. Define the logarithm of a number to a given base, and show how to change from one base to another. Why may not 1 be taken as base?

Given $\log_e x = 2\pi$, find x from the tables.

[Assume $\log_{10}\pi = \cdot 4971499$ and $\log_{10}e = \cdot 4342942$.]

8. Prove that in any triangle ABC

$$a^2 = b^2 + c^2 - 2bc\cos A.$$

Prove that, if Δ is the area of a quadrilateral whose sides are of lengths a, b, c, d, and θ the sum of a pair of its opposite angles, then

$$16\Delta^2 = 2\{b^2c^2 + c^2a^2 + a^2b^2 + a^2d^2 + b^2d^2 + c^2d^2\} \\ - (a^4 + b^4 + c^4 + d^4) - 8abcd\cos\theta.$$

9. Find formulæ for the solution of a triangle, given two angles and one side.

A man standing on one bank of a straight river sees two objects on the further side, and the lines joining his position to them make with the direction of flow of the river angles of 51° 36′ and 71° 48′. He walks down stream until the objects are seen in line, and finds that the line joining his position to them now makes an angle of 104° 57′ with the direction of flow of the river. He measures the distance he has walked, and finds it is 150 yards. What is the distance between the objects?

10. A cube standing upon a horizontal plane is cut by a plane which meets no vertical edge. Find expressions for the volumes cut off in the cases where (1) one of these edges is on one side of the plane and three on the other, (2) two of these edges are on one side of the plane and two on the other.

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V. STATICS AND DYNAMICS.

[*The measure of the acceleration due to gravity may be taken to be* 32 *when the foot and second are the units of length and time.*]

1. Explain what is meant by the resolved part of a force in a given direction.

If the resolved part is half the force, what is the angle between the given direction and that of the force?

2. Find the resultant of two parallel forces acting in opposite directions on a rigid body.

A heavy uniform rod ACB, of weight w, is supported, in a horizontal position, by a vertical string fastened to its middle point C; from the end B a weight w is suspended; and from a point D in CA a weight Q is suspended. Find Q, and the position of the point D, when the tension of the string at the middle point is equal to $4w$.

3. Find the position of the centre of gravity of a uniform triangular lamina.

In the sides AB, AC of a triangle ABC, points D, E are taken, such that AD is one-third of AB, and AE one-third of AC, and the points D, E are joined by a straight line; if F is the middle point of BC, and if G and H are the centres of gravity of ABC and of the quadrilateral $BDEC$, find the ratio of HG to AF.

4. Describe the common steelyard, and show how to graduate it.

If the length of the steelyard is four feet, if the resultant of the weights of the steelyard and scale-pan, which is equal to three pounds, acts in a vertical line distant sixteen inches from the end to which the scale-pan is attached, and if nineteen pounds is the greatest weight which can be measured when the movable weight is three pounds, find the position of the fulcrum.

5. Find the ratio of the power to the weight in the wheel and axle.

If two masses, each of weight w, are fastened to points on the wheel at an angular distance from each other of 120°, find the position of the wheel when the greatest possible weight is supported on the axle, and find what is this greatest weight.

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6. Explain in what sense a moving point may be said to have two velocities at the same time, and prove the rule for their composition.

A man in a railway train, travelling at a given rate, carries a loaded rifle, and fires it off so as to strike a telegraph post as he passes it; having given the velocity of the bullet, determine, by graphic construction, the direction in which he must aim.

7. Explain how uniform acceleration is measured.

How would the measure of the acceleration due to gravity be altered if the unit of time were half a second?

8. Two scale-pans, each weighing two ounces, are connected by a fine string passing over a smooth fixed pulley, and are hanging at rest. A weight of 15 oz. is placed in one scale-pan, and, simultaneously, a weight of 13 oz. is placed in the other scale-pan; find the acceleration of each scale-pan, the tension of the string, and the pressures of the weights on the scale-pans.

9. A stone is projected horizontally from the top of a tower, 256 feet in height, with the velocity of 128 feet per second; find when, and where, it will strike the horizontal plane on which the tower is situated, and the direction of its motion at that time.

10. If a point moves uniformly with velocity v in a circle of radius r, prove that its acceleration is in the direction of the centre of the circle, and is equal to v^2/r.

A heavy particle is attached to one end of a string three feet in length, the other is fastened to a fixed point; and the particle is held so that the string is inclined at the angle of 60° to the vertical. Find the velocity with which it must be projected horizontally so as to describe, uniformly, a horizontal circle.

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FURTHER EXAMINATION.

VI. PURE MATHEMATICS.

[*Full marks may be obtained for about* two-thirds *of this paper.*]

1. The perpendiculars drawn from the angular points A, C of the triangle ABC, to the opposite sides, cut these sides in D and F, and intersect one another in P. Prove that the tangents, at D and F, of the circle which passes through $BDPF$, pass through the middle point of the side CA; and that the tangents of this circle at P and B are parallel to CA.

2. If one side of a triangle be twice another side of this triangle, prove that the angle opposite to the former side is greater than twice the angle opposite to the latter side.

3. Prove that

$$(a^2+b^2+c^2)(a'^2+b'^2+c'^2)=(aa'+bb'+cc')^2+(bc'-b'c)^2+(ca'-c'a)^2+(ab'-a'b)^2.$$

4. What is the condition for the validity of the binomial expansion of $(1+x)^n$ in ascending powers of x?

Find the sum of the series

$$1+\frac{3!}{(1!)^2}\cdot\frac{4}{25}+\frac{5!}{(2!)^2}\left(\frac{4}{25}\right)^2+\frac{7!}{(3!)^2}\left(\frac{4}{25}\right)^3+\ldots \text{ to } \infty.$$

5. Prove that the amount of a sum of £A, in n years, at r per cent. per annum compound interest, payable yearly is

$$A\left(1+\frac{r}{100}\right)^n.$$

Hence find, with the help of the table of logarithms, the time in which a sum of money will double itself, at $2\frac{1}{2}$ per cent. per annum compound interest, payable quarterly.

6. Two persons, P, Q, stationed on a coast which runs East and West, observe a ship when it is due North of P, and again, when it is due North of Q. In the former case it is 45° West of North as seen from Q, in the latter case it is 30° East of North as seen from P. Determine the direction in which the ship is travelling.

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7. Mark the points (2, 2)(− 2, 2)(− 2, 0) with reference to the rectangular axes, OX, OY, and find the area of the triangle joining them.

8. Prove that the equation $ax+by=c$ represents a straight line.

Find the point of intersection of $3x+2y=10$ and $2x+3y=10$, and write down the equations of the straight lines which pass through this point and are each equally inclined to the two rectangular axes.

9. Find the equation of the chord of the circle $x^2+y^2=a^2$ which joins the points (x', y'), (x'', y''); and deduce the equation of the tangent of the circle at any point.

Show that the common tangents of the circles

$$x^2+y^2+2x=0, \qquad x^2+y^2-6x=0$$

form an equilateral triangle.

10. The ends A, B of a straight line AB, of constant length a, slide upon the fixed rectangular axes OX, OY respectively. If the rectangle $OAPB$ be completed, show that the coordinates of the foot of the perpendicular drawn from P to AB are $a\cos^3\theta$, $a\sin^3\theta$, where θ is the acute angle AB makes with OX.

11. Prove that if two parabolas have the same focus and axis, the locus of the point of intersection of two tangents at right angles, one to each of the parabolas, is a straight line.

12. Prove that if the pole of one straight line in regard to a conic lie upon another straight line, then the pole of the latter lies on the former.

If the straight lines intersect outside the conic, prove that they are harmonic in regard to the tangents of the conic drawn from their point of intersection.

13. Show how to draw tangents to a parabola from any exterior point.

Given the focus and a point of a parabola, and the tangent at the point, show how to describe the parabola.

14. Determine the radius of curvature at any point of an ellipse.

Tangents PQ, PR are drawn to an ellipse from a point P. Find the limit of the radius of the circle PQR as the point P approaches indefinitely near to, and ultimately lies upon, the ellipse.

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VII. MECHANICS.

[*Full marks will be given for about* two-thirds *of this paper. Great importance is attached to accuracy. Gravitational acceleration may be taken* = 32 *feet per second per second, and* $\pi = \frac{22}{7}$.]

1. If the directions and magnitudes of any forces in one plane be graphically represented by the sides AB, BC, CD, etc. of an unclosed polygon taken in order, prove that the direction and magnitude of the resultant is represented by the straight line closing the polygon.

If the polygon is closed, and the sides are the lines of action of the forces, determine whether the forces are in equilibrium.

A uniform heavy straight rod AB of one pound weight is placed with one end A resting on a rough horizontal plane and the other B against a smooth vertical wall. Supposing the rod to be on the point of motion, and the coefficient of friction to be $\frac{1}{2}$, determine by a graphical construction the magnitudes of the reactions at A and B, and the inclination of the rod to the horizon, giving the numerical results.

2. Prove that the sum of the moments of any two forces about any point in the plane of the forces is equal to the moment of their resultant about the same point.

Three forces P, Q, R act along the sides BC, CA, AB of a triangle ABC. Prove that their resultant passes through the orthocentre if

$$\frac{P}{\cos A} + \frac{Q}{\cos B} + \frac{R}{\cos C} = 0.$$

3. Show how to find the centre of mass (centre of gravity) of a quadrilateral area, and prove that three times the distance of that point from any straight line in the plane of the area is equal to the sum of the distances of the four corners minus that of the intersection of the diagonals from the same straight line.

A heavy quadrilateral area $ABCD$ is suspended from one corner A, M is the middle point of the diagonal BD, and O is the intersection of the diagonals. Prove that in equilibrium the vertical through A divides MO in V, so that

$$\frac{MV}{VO} = \frac{1}{2}\frac{CO}{OA}.$$

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4. A straight rod AB, whose weight can be neglected, is placed with its middle point C at the highest point of a fixed perfectly rough horizontal circular cylinder, the rod being at right angles to the axis. If two weights W, W' are suspended from the ends A, B, prove that the rod in its new position of equilibrium will be inclined to the horizontal at an angle

$$\frac{l}{a}\,\frac{W \sim W'}{W+W'},$$

where $2l$ is the length of the rod and a the radius of the cylinder.

How is the problem altered if the cylinder be not perfectly rough?

5. A system of n equal heavy pulleys, in which a separate string passes round each pulley with one end attached to a fixed horizontal beam and the other to the pulley next above (all the strings being vertical), has a power P acting upwards and a weight W downwards. Find the mechanical advantage.

Prove that the magnitude of the pull on the horizontal beam is $(P-w)(2^n-1)+nw$, where w is the weight of any pulley.

When there are five pulleys, prove that the distance of the point of application of the pull from the string of greatest tension is

$$\frac{26P-16w}{31P-26w}a,$$

where a is the radius of any pulley.

6. Particles whose masses are m_1, m_2, etc. are simultaneously projected on a smooth horizontal table with velocities V_1, V_2, etc. from a point O in directions making angles θ_1, θ_2, etc. with a fixed straight line Ox. Prove that their centre of gravity describes a straight line making an angle θ' with Ox where

$$\tan\theta' = \frac{\Sigma m V \sin\theta}{\Sigma m V \cos\theta}.$$

If two of the particles are connected together by a weightless slack string, determine how the motion of the centre of gravity is affected when the string becomes tight.

7. Find the time of descent from rest of a heavy particle on a smooth inclined plane.

Two heavy particles whose weights are W and W', slide down two smooth rods CA, CB in the same vertical plane, starting from rest at C. The rods are rigidly connected at C, which is fixed in space, and make angles θ and θ' with the vertical, on opposite sides of it. Prove that if the rods are held at rest, the centre of the circle circumscribing the particles and the point C moves vertically.

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Assuming the rods to have no weight, prove that no force, except that fixing C, is necessary to hold them at rest as the particles descend if

$$W \sin 2\theta = W' \sin 2\theta'.$$

8. Prove that the path of a heavy projectile *in vacuo* is a parabola, and that the square of the velocity varies as the distance of the particle from the focus.

Two particles are simultaneously projected from the same point A with equal velocities in directions AT, AT' where AT makes the lesser angle with the vertical, and the parabolas intersect in B. Determine which particle arrives first at B.

Prove also that the product of their times of transit from A to B is equal to the square of the time occupied by a particle falling from rest vertically through a distance equal to AB.

9. Two smooth imperfectly elastic spheres whose masses are m, m', impinge directly on each other with velocities V, V'. Find the velocities after impact.

Two smooth spheres moving in directions at right angles to each other impinge obliquely. If their directions of motion after impact are also at right angles, prove that the coefficient of elasticity is

$$\frac{m^2v - m'^2v'}{mm'(v - v')},$$

where m, m' are the masses and where v, v' are the components of the velocities of the spheres in the direction of the straight line joining the centres just before impact.

10. Explain what is meant by the work of a force. What is horse-power?

A given weight W is hoisted up to a given height h by a workman. During the first third of the height the weight is made to move upwards with a constant acceleration, during the second third with a constant velocity, during the last third it is brought gradually to rest with a constant retardation. Supposing that the resistances (including gravity) are represented by a given constant retarding force R, and that the whole time of the ascent is equal to that of falling from rest through a height nh, prove that the difference of the works done by the man in the first and third portions of the ascent is

$$\frac{25\,Wh}{18\,n}.$$

State also the whole work done.

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MATHEMATICAL EXAMINATION PAPERS

FOR ADMISSION INTO

Royal Military Academy, Woolwich,

NOVEMBER, 1898.

I. PART I. FIRST PAPER.

[*Great importance will be attached to accuracy.*]

1. Reduce to its simplest form

$$\frac{(8\tfrac{4}{5}+\tfrac{1}{9}-\tfrac{1}{45})(4\tfrac{1}{5}-3\tfrac{1}{4})}{(1\tfrac{5}{11}+\tfrac{1}{8})-(\tfrac{9}{16}-\tfrac{1}{8}-\tfrac{1}{22})}.$$

2. Find the value of £1·875 − 2·25 crowns + 3·75 guineas − 46·125 shillings.

3. At what rate of simple interest must £230,398. 6*s*. 8*d*. be invested in order that it may amount to £298,941. 16*s*. 9*d*. at the end of seven years?

4. Of a certain store of potatoes, 11 men would in 3 days consume all but 201 pounds, and 21 men would in 4 days consume all but 48 pounds; how many pounds of potatoes does the store contain?

5. If an integer which is less than 50 become a square number when 25 is added to it, show that it can be resolved into two factors whose difference is 10, and hence show that in such an integer the units digit is equal to the square of the tens digit; write down all such integers.

6. Find the value of

$$\frac{3(x+y+z)(yz+zx+xy)-x^3-y^3-z^3}{x^2+y^2+z^2-yz-zx-xy}$$

when $x=\frac{1}{2}$, $y=\frac{2}{3}$, $z=\frac{3}{4}$.

7. Prove that the remainder which is left when x^4+7x+3 is divided by $x-1$ is the same as the remainder which is left when x^4-6x+7 is divided by $x-2$.

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8. In any division sum show that a common factor of the divisor and the dividend is a factor of any of the successive complete remainders.

In finding the highest common factor of two expressions, $2x^4 - x^3 - 3$ was one of the divisors, and $5(x^3 + 3x + 4)$ was the corresponding remainder. What was the highest common factor of the two expressions?

9. Solve the equations

(i.) $$\frac{(a^2-1)(ax+1)}{a^2(x+a)} + \frac{(a^2+1)(x-a)}{ax+1} = \frac{ax+1}{x+a} + \frac{a(ax-1)}{ax+1};$$

(ii.) $$\begin{cases} \frac{3}{2}(x-y) + \frac{1}{6}(x+2y) = 4 \\ 2(x-y) + 3(x+2y) = 19; \end{cases}$$

(iii.) $$\frac{7}{x-1} + \frac{5}{x-2} + \frac{22}{3}\,\frac{1}{x-3} = 0.$$

10. The mth and n^{th} terms of an arithmetical progression are p, q. Find the first term and the common difference.

The thirty-first term of a certain arithmetical progression exceeds the square of one-half of the fourth term by unity, and also exceeds twice the fourteenth term by unity; find its values.

11. Prove that the number of combinations of n things taken r at a time is

$$\frac{n!}{(n-r)!\,r!}.$$

You have seven envelopes directed to seven people, to four of whom you intend to send copies of a circular, the other three envelopes to be used for a different purpose. In how many incorrect ways can you put the four circulars in four of the seven envelopes?

12. Assuming that the binomial theorem is true for a positive integral exponent, prove that it is true for a positive fractional exponent.

Apply the binomial theorem to find the fifth root of 3126 to seven places of decimals.

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II. PART I. SECOND PAPER.

N.B.—*In questions on Geometry ordinary abbreviations may be employed, but the method of proof must be geometrical. Proofs other than Euclid's must not violate Euclid's sequence of propositions. In the absence of special directions to Candidates, any of the propositions within the limits prescribed for Examination may be used in the solution of problems and riders.*

1. Show that, if from the ends of the base of a triangle two right lines are drawn to any point within the triangle, the sum of these is less than the sum of the other two sides of the triangle, but they contain a greater angle.

Show that, if any point inside a triangle is joined to the three vertices, the sum of the joining lines is less than the sum, and greater than half the sum, of the three sides of the triangle.

2. Show that, if the square on one side of a triangle is equal to the sum of the squares on the other two sides, the triangle is right-angled.

Explain the meaning of the *converse* of a theorem. What is the converse of the theorem in this question?

3. Assuming Propositions 12 and 13 of Euclid, Bk. II., prove that the sum of the squares on the diagonals of any parallelogram is equal to the sum of the squares on the sides.

4. Prove that all angles contained in the same segment of a circle are equal, those in a semicircle being right angles.

If any two circles be drawn each touching the three sides of a triangle, prove that the circle described on the line joining their centres as diameter passes through two vertices of the triangle.

5. Show how to draw a tangent to a circle from any given external point.

Two points are given; one is to be the centre of a circle, and the tangent drawn to this circle from the other is to be of given length less than the distance between the given points; show how to draw the circle and the tangent.

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6. Show how to describe a square about a given circle.

Every parallelogram described about a circle must have all its sides equal.

7. Define similar triangles, and state (without proof) the relation between their areas.

Give any method by which an equilateral triangle may be inscribed in a given triangle ABC, having one of its sides perpendicular to AB.

8. There is a piece of ground in the form of a trapezium, the lengths of the parallel sides of which are 20 and 34 yards, and the lengths of the other two sides 15 and 13 yards; find its area.

9. Prove that the area of the curved surface of a frustum of a cone is equal to the slant height multiplied by the perimeter of the mid section.

10. Find the number of cubic feet of earth removed per yard length from a cutting in level ground 12 feet in depth, the breadth of the base of the cutting being 15 feet, and the slopes of both sides 45°.

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III. PART I. THIRD PAPER.

1. Prove that the angle subtended at the centre of a circle by an arc which is equal in length to the radius is an invariable angle, and explain what is meant by the circular measure of an angle.

A circular wire of 3 inches radius is cut, and then bent so as to lie along the circumference of a hoop whose radius is 4 feet. Find the angle which it subtends at the centre of the hoop.

2. Define the *cotangent* and *cosecant* of an angle, and show that

$$\cot^2 A + 1 = \operatorname{cosec}^2 A.$$

Which is greater, the acute angle whose cotangent is $\frac{4}{3}$, or the acute angle whose cosecant is $\frac{5}{4}$?

3. Obtain an expression for all the angles which have a given tangent.

Find all the angles, lying between $-360°$ and $+360°$, which satisfy the equation

$$\tan^2 x - \frac{2}{\sqrt{3}} \tan x - 1 = 0.$$

4. Prove geometrically, for the case in which A and B are two positive angles whose sum is less than a right angle, that

$$\sin(A+B) = \sin A \cos B + \cos A \sin B.$$

Express $\dfrac{\sin 3A}{\sin 2A - \sin A}$ in terms of $\cos A$.

5. Prove the formulæ

$$(1+\cos A)\tan^2 \tfrac{1}{2}A = 1 - \cos A$$

$$(\sec A + 2\sin A)(\operatorname{cosec} A - 2\cos A) = 2\cos 2A \cot 2A.$$

6. In any triangle ABC, show that

$$\sin \tfrac{1}{2}A = \sqrt{\frac{(s-b)(s-c)}{bc}}.$$

Find the greatest angle of the triangle whose sides are 184, 425, and 541.

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7. Define the logarithm of a number, and point out the use of logarithms in arithmetical calculations.

From
$$\log 2 = 0{\cdot}3010300$$
$$\log 3 = 0{\cdot}4771213$$
$$\log 11 = 1{\cdot}0413927$$

find the values of $\log 7{\cdot}92$, $\log 0{\cdot}001089$, and $\log\sqrt{\frac{11}{12}}$.

8. If ABC be a triangle, and θ such an angle that $\sin\theta = \frac{2\sqrt{ab}}{a+b}\cos\frac{1}{2}C$, find c in terms of a, b, and θ.

If $a = 11$, $b = 25$, and $C = 106^\circ\ 15'\ 37''$, find c, having given

$L\cos 53^\circ\ 7'\ 48''{\cdot}5 = 9{\cdot}7781509$;
$L\sin 33^\circ\ 33' \quad = 9{\cdot}7424616$. Tab. Diff. 1904.
$L\cos 33^\circ\ 33' \quad = 9{\cdot}9208555$. Tab. Diff. 838.

For other logarithms see Question 7.

9. If tangents be drawn to the inscribed circle of a triangle parallel to the sides of the triangle, show that the areas of the triangles cut off by these tangents are inversely proportional to the areas of the corresponding escribed circles.

10. Prove that

$$\log_e(x+1) - \log_e x = 2\left\{\frac{1}{2x+1} + \frac{1}{3(2x+1)^3} + \frac{1}{5(2x+1)^5} + \dots\right\},$$

and deduce that

$$\log_e 13 = 2\log_e 2 + \log_e 3 + {\cdot}0800427\dots.$$

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IV. PART II. FIRST PAPER.

[*Full marks may be obtained for about* three-quarters *of this paper.*]

1. Solve the simultaneous quadratics

$$\text{(i.)}\quad \begin{cases} x^2+xy=10, \\ 2xy-y^2=3\,; \end{cases}$$

$$\text{(ii.)}\quad \begin{cases} x^2+y^2=13, \\ 2x-xy+2y=4. \end{cases}$$

Give the general rule for solving when the unknown quantities occur symmetrically in the equations as in (ii.).

2. Give the mth term in the expansion of

$$(1-x)^{\frac{1}{n}}$$

by the binomial theorem.

Prove that

$$\frac{1}{2}\sqrt[5]{28}=\left(1-\frac{1}{8}\right)^{\frac{1}{5}}=1-\frac{1}{40}-\sum_{m=2}^{m=\infty}\frac{1\cdot 4\cdot 9\ldots(5m-6)}{\lfloor m}\left(\frac{1}{40}\right)^m.$$

3. Prove the identity

$$\sec 2A = 1+\tan A\tan 2A.$$

Solve the equation

$$\sin(\alpha+x)+\sin(\beta+x)=0.$$

4. Find the area of a regular quindecagon inscribed in a circle of one foot radius, making use of the table of natural sines.

5. Inscribe an equilateral and equiangular pentagon in a given circle.

In the isosceles triangle ABC, whose base is BC, each base angle is double of the third angle. In AB take a point D such that CD bisects the angle ACB. Prove that BC is equal to the side of a regular pentagon inscribed in the circle ADC.

6. Prove that if a straight line be drawn parallel to one of the sides of a triangle it cuts the other sides or those sides produced proportionally.

In the side BC of a triangle ABC take a point D and draw DE, DF parallel to CA and BA respectively to meet AB in E and AC in F. Find the locus of the middle point of EF.

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7. Show that, if a solid angle be contained by three plane angles, any two of them are greater than the third.

8. Prove that similar polygons inscribed in circles are to one another as the squares on the diameters of the circles.

9. Define the latus rectum of a conic section, and show that the tangents to the curve at its extremities intersect in the directrix.

Prove that the focal distance of a point on a conic is equal to the length of the ordinate produced to meet the tangent at the end of the latus rectum.

10. In a conic, SP is the focal distance of any point P, and PG is the normal. Prove that the ratio of SG to SP is constant.

Prove that if normals be drawn at the ends of a focal chord, a line through their intersection parallel to the transverse axis will bisect the chord.

11. P is any point on a conic, and T any point in the tangent at P. If TM be drawn perpendicular to the focal distance SP and TN perpendicular to the directrix, show that SM is to TN in a constant ratio.

PQ is any chord of a conic, subject to the condition that it subtends a constant angle at the focus, and the tangents at P and Q intersect at T. Show that the locus of T is a conic with the same focus and directrix.

12. If the straight line joining two points P, P' of a conic meet a directrix in F, and F be joined to the corresponding focus S, prove that FS will bisect one of the angles between PS and $P'S$.

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V. PART II. SECOND PAPER.

[*Full marks may be obtained for about* three-quarters *of this paper. Great importance is attached to accuracy. Gravitational acceleration may be taken* = 32 *feet per second per second.*]

1. Obtain the components of a given force parallel and perpendicular to a given line.

A body which weighs 12 lbs. is kept at rest by means of two cords, one being horizontal, and the other inclined to the horizontal at an angle whose tangent is $\frac{3}{4}$; find their tensions.

2. On the two smooth fixed rods AB and AC, equally inclined to the vertical at an angle of 30°, slide two small rings each of weight 2 ounces. Another weight of 10 ounces is knotted at E to the light thread DEF, length 4·6 inches, and the ends D and F are tied to the rings. Find graphically the directions of the two parts of the thread, and the position in which it hangs between the rods.

Given $DE : EF = 1 : 2$.

3. Define the moment of a force about a point, explaining the rule of signs.

Prove that the sum of the moments of two parallel forces acting on a rigid body about any point in their plane is equal to the moment of their resultant about the same point.

4. Define the centre of mass of a body.

A uniform lever of weight 6 lbs. and length 18 inches has weights of 11 lbs. and 7 lbs. attached at its ends; find the centre of mass of the system.

5. State the conditions of equilibrium of a system of forces acting in one plane on a rigid body.

A rigid uniform bar of weight 10 lbs. rests with its lower end in contact with a horizontal rough plane and makes an angle of 30° with the horizontal, being kept in position by a horizontal thread attached to its upper end. Find the tension of the thread and the magnitude and direction of the force exerted at the lower end.

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6. A body of given weight W lbs. rests on a smooth plane of inclination α to the horizontal under a force acting up the plane. Find the magnitude of the force and the pressure exerted by the plane on the body.

If the body is a uniform cube with four of its edges horizontal and one face in contact with the plane, and if the force applied to support it acts in a line of greatest slope in that face, find the least inclination of the plane for which the cube will turn about its lowest edge.

7. A pistol shot is fired from a railway carriage travelling with velocity u so as to strike an object seen (at the instant of firing) in a line at right angles to the direction of motion of the carriage. Assuming that the charge of powder in the cartridge can impart a velocity v to the shot, find the direction in which the pistol must be aimed.

8. Define the acceleration of a point moving in a straight line.

Prove that if the point moves with uniform acceleration its average velocity in any interval of time is half the sum of its velocities at the beginning and end of the interval.

Two particles move from the same point A along the same line AB, one of them having a uniform velocity u, and the other a uniform acceleration f and no initial velocity. Find the time that elapses before the second overtakes the first.

9. A body weighing W lbs. is carried up in a lift which moves with an acceleration of 4 ft. per sec. per sec. State the direction of the force exerted upon it by the floor of the lift, and the weight of a body which this force could support at rest.

How would the result be modified if the lift ascended with uniform velocity?

10. A body weighing W lbs. is acted on by a force which could support a weight of W' lbs. at rest. In what time will it acquire a velocity v feet per sec.?

A body is suspended by a cord from the roof of a railway carriage forming part of a starting train, and it is observed that the cord is inclined to the vertical at an angle whose tangent is $\frac{1}{8}$. In what time will the train acquire a velocity of 30 miles an hour?

11. Determine the magnitude and direction of the acceleration of a point describing a circle of radius a with uniform speed v.

A wheel of radius r is rolling uniformly with velocity v along a level road. State the magnitude and direction of the velocity and acceleration (a) of the point instantaneously highest, (b) of the point instantaneously lowest.

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12. A shot is fired from the edge of a vertical cliff of height b to strike an object at a distance a from the foot of the cliff. Prove that, if the velocity of projection is that due to falling through a height h, the angle of elevation α is given by the equation

$$a^2 \tan^2\alpha - 4\,h\,a \tan\alpha + a^2 - 4\,h\,b = 0,$$

and hence find the condition that the object may be within range. The resistance of the air is to be neglected.

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VI. PART III. FIRST PAPER.

[*Full marks will be given for about* two-thirds *of this paper.*]

1. Prove geometrically that the subnormal in the parabola is equal to the semi-latus rectum.

PQ is a chord perpendicular to the axis, and QR is drawn perpendicular to the tangent at P to meet the diameter through P in R. Prove that PR is equal to the latus rectum.

2. C is the centre and A the vertex of an ellipse. The ordinate and tangent at any point P meet the transverse axis in N and T. Prove that $CN \cdot CT = CA^2$.

Show that any circle passing through N and T cuts the auxiliary circle orthogonally.

3. If perpendiculars to the asymptotes be drawn from any point on a rectangular hyperbola, prove that their product is constant.

AB and AC are fixed straight lines at right angles to one another, and the straight line BOC passes through a fixed point O, the points B, C being otherwise undetermined. Prove that the locus of the middle point of BC is a rectangular hyperbola, and determine its asymptotes.

4. Form the equations of the perpendiculars of the triangle whose sides are

$$a_1x + b_1y + c_1 = 0,$$
$$a_2x + b_2y + c_2 = 0,$$
$$a_3x + b_3y + c_3 = 0.$$

Deduce that they are concurrent, and show how to find the coordinates of the orthocentre.

5. Investigate the equation to the tangent to the parabola in the form

$$y = mx + \frac{a}{m}.$$

Show that the parabolas

$$y^2 = 4ax, \qquad x^2 = 4by$$

intersect at an angle of 30° if

$$2(a^{\frac{2}{3}} + b^{\frac{2}{3}}) = 3\sqrt{3}a^{\frac{1}{3}}b^{\frac{1}{3}}.$$

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6. Find the lengths of the perpendicular from the foci of an ellipse on the tangent at the point whose eccentric angle is ϕ.

Show that they may be obtained by solving the equation

$$u^2 - \frac{2ab}{b'}u + b^2 = 0$$

where a, b are the semi-principal axes, and b' the semi-diameter conjugate to the central vector of the point.

7. Define the hyperbola conjugate to a given hyperbola.

Investigate the equation of the hyperbola conjugate to the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1.$$

8. State the circumstances under which a heavy body will rest on a rough inclined plane without toppling over.

A heavy uniform square lamina balances in a vertical plane on the highest point of a fixed vertical circle in the same plane. The circular rim being rough enough to prevent sliding, show that, if the side of the square be $\frac{\pi r}{\sqrt{3}}$, it can rock through an angle of 60° without falling; r being the radius of the circle.

9. Describe the mechanical power known as the lever. A heavy uniform bar has centre C and one end D. It is placed across two pegs A, B, which are in the same horizontal straight line, in such a manner that they divide the distance CD symmetrically, and it is kept in equilibrium by a weight P suspended from D. If the largest possible value of P is λ^2 times the smallest possible value, show that the distance between the pegs is $\frac{1}{2} \cdot \frac{\lambda - 1}{\lambda + 1}$ of the length of the bar.

10. At a given instant a particle is sliding down a rough plane, inclined at an angle α to the horizon, with a velocity of 32 feet per second; the coefficient of friction being μ, determine the nature of the subsequent motion.

If $\mu = \frac{2}{3}$ and $\alpha = 30°$, for how many seconds will it move before coming to rest?

11. If a ball be projected in vacuo, show that the velocity at any point is that which it would acquire if dropped freely to that point from the directrix to its path.

A ball is projected and a second ball also from the same point, and in the same direction, and with a velocity equal to the vertical velocity of the first ball. Prove that the path of the second passes through the focus of the path of the first.

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12. A weight W lbs. is raised from rest through a vertical height H feet in T seconds, when it again comes to rest, by means of a chain which sustains a uniform tension for a part of the time and then becomes slack. Show that the weight ascends with acceleration

$$\frac{2gH}{gT^2 - 2H} \quad \text{for} \quad \frac{gT^2 - 2H}{gT} \text{ seconds.}$$

Further show that if the safe tension of the chain be P lbs. weight the time T must not be less than

$$\frac{v}{g}\sqrt{\frac{P}{P-W}} \text{ seconds}$$

where v is the velocity due to the height H.

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VII. PART III. SECOND PAPER.

[*Full marks will be given for about* two-thirds *of this paper.*]

1. Show how to draw two tangents to a parabola from a given external point.

Prove that, if three tangents form a triangle PQR, and p is the point of contact of QR, and S the focus, then

$Qp : Rp = PR \,.\, SQ : SR \,.\, PQ$, the point p being between Q and R.

2. Prove that, if CP, CD are conjugate semi-diameters of an ellipse whose foci are S and S', then $SP \,.\, S'P = CD^2$.

Prove also that, if the tangents at P and D meet in T, and if SP is produced to R so that $PR = SP$, then the triangles STR, SDT are similar.

3. Define the asymptotes of an hyperbola.

Prove that, if the tangent at P meets one asymptote in T, and the focal radius SP meets the same asymptote in U, the triangle STU is isosceles.

4. Find the angle between the two straight lines whose equations are

$$y = mx + b \quad \text{and} \quad y = m'x + b'.$$

Form the equation of the straight line drawn through the point $x = 1$, $y = 2$ at right angles to the line $x + 2y = 3$.

5. Find the equation of the circle which has a given centre and radius.

Prove that the locus of a point from which the tangents to two given circles are in a constant ratio is a circle.

6. Find the equation of the locus of the middle points of the chords of the parabola $y^2 = 4ax$ which make an angle θ with the axis of x.

Prove that the locus of the middle points of chords of $y^2 = 4ax$, drawn through a given point, is a parabola, and explain the connexion between the two results.

7. Find the equation of the normal at any point (x', y') on the ellipse $x^2/a^2 + y^2/b^2 = 1$.

Prove that the normals at the extremities of the chords $lx/a + my/b = 1$ and $x/la + y/mb = -1$ meet in a point, and find its co-ordinates.

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8. Define the moment of a force about a line.

A rigid uniform triangular board is supported by three equal vertical strings whose upper ends are attached to the corners of an equal triangle fixed in a horizontal plane, and a sphere of given weight is placed at a given point on the board. Determine the tensions of the strings.

9. Explain what is meant by the *coefficient of statical friction* between two bodies.

A rigid uniform bar rests against a vertical wall and a horizontal floor, the vertical plane containing the bar being at right angles to the wall. Prove that, if the bar is on the point of slipping down, the angle α which it makes with the horizontal is given by the equation

$$\tan \alpha = \tfrac{1}{2}(1/\mu' - \mu)$$

where μ and μ' are the coefficients of friction at the upper and lower extremities.

10. A smooth sphere of mass m moving with velocity u impinges directly on a smooth sphere of mass m' at rest. Find the velocities of the spheres after impact, the coefficient of restitution being $\frac{1}{4}$.

Prove that, if I is the magnitude of the impulse between the spheres, the kinetic energy lost in the impact is $\frac{3}{8} Iu$.

11. Prove that a particle moving freely under gravity describes a parabola.

If the particle is projected at an elevation α and strikes an inclined plane through the point of projection at right angles, the inclination θ of the plane to the horizontal is given by the equation

$$\tan(\alpha - \theta) = \tfrac{1}{2} \cot \theta.$$

12. Two particles of equal mass are connected by an inelastic string, and held near to each other on a smooth horizontal table. A ring of mass equal to that of either particle is threaded on the string and hangs just over the edge of the table. Prove that, when the particles are let go, the ring descends with acceleration $\frac{1}{3}g$.

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MATHEMATICAL EXAMINATION PAPERS

FOR ADMISSION INTO

Royal Military Academy, Woolwich,

JUNE, 1899.

I. PART I. FIRST PAPER.

[*Great importance will be attached to accuracy.*]

1. Reduce to their simplest forms: $\frac{1}{2}+\frac{2}{3}+\frac{3}{4}$, $2\frac{3}{5}-\frac{9}{20}$, and $2\frac{3}{7}-\frac{34}{101}$, and divide the product of these three expressions by $\frac{13}{4}+\frac{29}{30}-2$.

2. A tradesman, having promised me $12\frac{1}{2}$ per cent. reduction from a bill for £16. 16*s*. 8*d*. takes off only 10 per cent. By what sum am I entitled to further diminish his bill? And how much per cent. is this of the account rendered?

3. Define the *average* of a set of numbers, and find the average of 213, 217, 199, 201, 208, 209, and 211.

If the height of a number of men is measured; and the average of the first six is 5 ft. 5 in., of the next seven, 5 ft. $5\frac{1}{2}$ in., and of the next eleven, 5 ft. $4\frac{3}{4}$ in.; what is the average height of the whole number of men to the nearest hundredth of an inch?

4. Two trains start at the same time, one from Liverpool to Manchester, and the other in the opposite direction, and running steadily complete the journey in 42 minutes and 56 minutes respectively. How long is it from the moment of starting before they meet?

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5. A certain number is the product of 12 and another factor. The number is divisible by 16 and by 17. Show that the second factor is divisible by 17, but not necessarily by 16.

State a general proposition of which this is a particular case.

6. Multiply together any three consecutive whole numbers, all greater than 100. Add to this product the middle number, and show (by extracting the cube root or otherwise) that the result is a perfect cube.

If a is the smallest of the numbers, state and prove the result just given in a general form.

7. Divide $x^7 - 729x$ by $x^2 - 3x + 9$, and express the quotient in its simplest real factors.

8. Solve the equation $\frac{x-1}{5} = 8 - \frac{x+1}{3}$.

Also if the square of the first side of the above equation is equal to the second side (unaltered), find the *two* values of x; and explain why the first equation has only one solution and the second two.

9. Find the values of x and y in terms of a and b from the equations

$$x - \frac{y}{2} = \tfrac{1}{2}\left(a - \frac{b}{2}\right);$$

$$x + y = \tfrac{1}{2}(a + b - 1).$$

If the values of x and y which are to satisfy this equation are 50 and 51, what must be the values of a and b?

10. An arithmetical and geometrical progression have the same first term 10; and the common difference of the former series, which is 7, is equal to the common ratio of the latter series. Find if any of the numbers 3430, 3455, 3475 belong to either of these series, and, if so, to which.

If $-a$ and $+a$ are two terms in an arithmetic progression, and the number of intermediate terms is $2n$, find the value of the two terms which are numerically smallest.

11. Find the number of permutations of four different things taken all together, giving your *reasons* in full.

A boy, fresh from school, boasts that he is able to distinguish the four brands of champagne (of which he knows the names) in his father's cellar. Accordingly his father fills him a glass of each wine without allowing him to see the labels on the bottles, and requires him to name each variety.

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The boy gives the four names and gives each wrong. Determine in how many different ways he can do this, and in how many ways he could have named at least one right.

12. Prove the Binomial Theorem for a positive integral index.

Employ it to show that, to the nearest pound, the interest on £10,000 for 10 years at 2 per cent. compound interest per annum is £2190.

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II. PART I. SECOND PAPER.

N.B.—*In questions on Geometry ordinary abbreviations may be employed, but the method of proof must be geometrical. Proofs other than Euclid's must not violate Euclid's sequence of propositions. In the absence of special directions to Candidates, any of the propositions within the limits prescribed for Examination may be used in the solution of problems and riders.*

1. Name and define the different four-sided figures of which Euclid takes note.

If the middle points of every pair of adjacent sides of a rhombus be joined, what is the four-sided figure so formed? Prove that your answer is correct.

2. Through a given point draw a straight line parallel to a given straight line.

Construct a triangle, having given one angle and the lengths of the perpendiculars from the other two angles on the opposite sides.

3. If a straight line be divided into two equal parts, and also into two unequal parts, prove that the rectangle contained by the two unequal parts, together with the square on the line between the points of section, is equal to the square on half the line.

Compare the lengths of the two unequal parts, if the rectangle contained by them is eight times the square on the line between the points of section.

4. Draw a tangent to a circle from an external point.

Given a circle and a straight line, find a point on the line such that, if tangents be drawn to the circle, the chord of contact will subtend a given angle at the circumference. Is it always possible to solve this problem? Give your reasons.

5. A chord AB of a circle is produced to C, so that BC is 7 inches in length. The tangent from C is one foot in length. What is the length of AB?

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6. In a given circle place a straight line equal to a given straight line which is not greater than the diameter of the circle.

Show that the straight line subtends an angle of 30° at the circumference if the given straight line be equal to the radius.

7. Find in what ratio the bisector of any angle of a triangle divides the opposite side.

The edge of your paper prevents you from completing *one* angle of a triangle. Show how to draw as much of the internal and external bisectors of that angle as will appear upon your paper, noting carefully what points in your construction you assume to be within the limits of your paper.

8. With a pair of scissors a square, 36 square inches in area, is divided by a straight cut through one angle and the middle point of an opposite side. Find the areas of the two portions.

If another straight cut is made through two opposite angles, so as to cross the first cut, find the areas of the four portions.

9. An equilateral triangle, 10 inches in the side, revolves about one side. Find to the nearest cubic inch the volume of the solid generated (*i.e.*, the volume within the surface traced out by the other two sides).

10. If the area of one face of a regular tetrahedron is A, what is its volume?

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III. PART I. THIRD PAPER.

1. Define the *radian*.

The sun is 93 million miles distant, and subtends an angle of 0·00466 of a radian. Find its diameter.

2. Prove the relation $\sin^2\theta + \cos^2\theta = 1$, and show that the fractions $\sin\theta/(1-\cos\theta)$ and $\sin\theta/(1+\cos\theta)$ are reciprocal. (The sign / denotes the fraction-line.)

Hence find a third proportional to $1+\cos\theta$ and $\sin\theta$, when θ approaches indefinitely near to two right angles.

3. Prove, by means of a geometrical figure, that $\sin 2\theta = 2\sin\theta\cos\theta$, 2θ being an acute angle.

If Q is the quotient obtained by dividing $\sin^3\theta + \cos^3\theta$ by $\sin\theta + \cos\theta$, prove that $\sin 2\theta = 2(1-Q)$.

4. Ascertain and write down a general formula giving all angles, both positive and negative, of which the cosine is equal to $\cos A$.

Find the positive values of the angle A, less than 360°, for which $\cos(A+20°) = \cos(3A+60°)$.

5. The cosecant of an acute angle is $\frac{5}{4}$. What are its sine, cosine, and tangent? What changes in the sine, cosine, and tangent occur, if the angle is not acute but obtuse?

Considering acute angles only, prove that the angle

$$\operatorname{cosec}^{-1}\frac{5}{4} - \cot^{-1}4$$

is less than half a right angle.

6. Prove that in any triangle $\cos^2\frac{A}{2} = \frac{s(s-a)}{bc}$.

Find the cosine of half the greatest angle, and the sine of the least angle in a triangle of which the sides are 109, 91, 60.

7. Find an expression for the radius of the inscribed circle of a triangle in terms of the sides.

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If the sides are 3, 4, and 5 inches in length, in what ratios do the points of contact divide the sides?

8. Show how to solve a triangle, having given two sides and the angle opposite one of them. Determine the condition that the triangle may exist (i.) when the given angle is acute, (ii.) when it is obtuse.

If $a = 1000$ inches, $b = 353$ inches, $B = 20° \; 35'$, find, with the aid of the tables supplied, the angles A and C, taking A to be obtuse.

9. Write down the logarithms of the numbers 1234, 2345, 345·1, 45·12, 5·123; and calculate, as accurately as your tables permit, the value of the fraction

$$\frac{1234 \times (2345)^2 \times (345 \cdot 1)^3}{\sqrt[4]{45 \cdot 12} \times \sqrt[5]{5 \cdot 123}}.$$

10. Define the logarithm of a number to the base x.

What is the relation between two numbers, if the sum of their logarithms to base x is (1) zero, (2) unity, (3) three times the lesser of the two logarithms?

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IV. PART II. FIRST PAPER.

[*Full marks will be given for about* three-quarters *of this paper.*]

1. Find the condition that the roots of the quadratic equation

$$ax^2 + 2bx + c = 0$$

may be real and distinct.

If α and β are the roots of the above equation, form the equation whose roots are $\alpha + 1/\beta$ and $\beta + 1/\alpha$.

2. Find the number of sets of r things which can be formed with n different things.

Prove that the number of ways in which nine different things can be distributed into sets of three is 280.

3. Prove that the angle subtended at the centre of a circle by any chord is double of the angle in the greater segment cut off by the chord.

If AB is the chord, C the centre of the circle, P any point on the greater segment cut off by AB, and if PA and PB meet the diameter perpendicular to AB in D and E, then the triangles DPE and DCA are similar.

4. On the sides AB and AC of a triangle ABC points F and E are taken so that $CE : EA = AF : FB = 2 : 1$. Find the ratio of the areas of the triangles ABC and AEF.

Also prove that, if L and M are the feet of the perpendiculars let fall on BC from E and F, then $EL : FM = 2 : 1$.

5. The base BC of a triangle ABC is divided at D so that

$$BD : CD = n : 1.$$

Prove that $(n + 1) \cot ADC = n \cot BAD - \cot DAC$.

6. A church spire is seen in a direction due North of a station A at an elevation of $17°$, and from a station B 120 feet due East of A the spire bears $23°$ West of North. The two stations and the foot of the spire being at the same level, determine the height of the spire.

7. Prove that a straight line which is perpendicular to each of two straight lines at their point of intersection is perpendicular to the plane containing them.

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8. Prove that, if each of the edges of a tetrahedron is perpendicular to the opposite edge, the lines joining the middle points of pairs of opposite edges are equal in length.

9. Define a conic section by means of a focus and directrix.

Prove that the line joining the focus to the point of intersection of any chord with the corresponding directrix is one of the bisectors of the angle between the radii drawn from the focus to the extremities of the chord.

10. Prove that the semi-latus rectum of a conic is a harmonic mean between the segments of any focal chord.

11. Prove that the tangent at P to a conic whose focus is S and the line through S perpendicular to SP meet on the directrix.

If R is the point where they meet, SY perpendicular to the tangent PR, Q a point where SR meets the conic, and QK parallel to PR meets the directrix in K, then $PY : SP = SQ : QK$.

12. Prove that the projection on the focal radius of the normal terminated by the transverse axis is equal to the semi-latus rectum.

Given a point of a conic, the tangent at the point, a focus, and the length of the semi-latus rectum, construct the direction of the transverse axis.

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V. PART II. SECOND PAPER.

[*Full marks may be obtained for about* three-quarters *of this paper. Gravitational acceleration may be taken equal to* 32 *feet per second per second.*]

1. State the theorem known as the parallelogram of forces.

A uniform cubical block of wood of weight W rests on a smooth inclined plane with four of its edges parallel to the line of greatest slope, and another of its edges in contact with a smooth vertical wall. If the inclination of the plane be 45°, find the pressure on the wall.

2. Define a couple and its moment.

Show that two couples in the same plane are in equilibrium if their moments are equal and opposite.

3. AB and BC are two uniform beams fixed together at B of length 3·72 and 1·92 inches respectively. AB rests on a horizontal plane, and BC lies in the vertical plane containing AB, the angle ABC being 125°. Find graphically the line of the resultant reaction between AB and the plane. The weights of the beams are proportional to their lengths.

4. Find the centre of mass of two bodies, the mass and centre of mass of each being given.

Three weightless wires form a triangle ABC, which is suspended from A. Masses of 2 lbs. and 1 lb. are fixed to the middle points of AB and AC, and one of 3 lbs. to the corner C. What mass must be placed at B so that in equilibrium the point of trisection of BC next B shall be vertically under A?

5. A balance consists of a perfectly uniform isosceles triangle suspended from the vertex, the scale-pans being hung from the ends of the base. Unequal weights are placed in the pans and balance. They are interchanged, and to effect equilibrium one ounce has to be added to one of them. Show that the one scale-pan is half an ounce heavier than the other.

6. What is meant by the angle of friction?

A rough triangular prism is fixed with one face on a horizontal plane. The other faces are inclined to the plane at angles of 55° and 20°. Masses m_1, m_2 are connected by a thread passing over the upper edge of the prism and lying along lines of greatest slope, and rest on the two inclined faces. If the coefficient of friction on each face is tan 25°, find the ratio $m_1 : m_2$ when the equilibrium is critical.

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7. Three points are situated at the corners of an equilateral triangle. They all start with velocity V along the sides in the same direction round the triangle. Find the velocity of any one relative to any other.

8. Give Newton's Second Law of Motion.

On what units does the unit of force depend? The foot, second and pound being the units of length, time and mass, how many units of force are there in the weight of one pound?

What experimental result shows that the weights of bodies are proportional to their masses?

9. A balloon is ascending with uniform acceleration equal to $\frac{1}{8}g$. Find the weight indicated by a spring balance in the balloon-car, on which a mass of one pound is hung.

10. What is a horse-power?

At what horse-power is an engine working which is dragging a load of six tons up an incline of one in ten at a rate of fifteen miles per hour? (Friction and all retarding forces except gravity to be omitted.)

11. A cone of semi-vertical angle 30° is fixed with its axis vertical, and vertex upwards. A bead is attached to the vertex by a thread of length two feet, and rests on the smooth surface of the cone. What velocity must be given to it that it may move round the cone in a horizontal circle without causing any pressure on the surface?

12. Two particles, moving with equal velocities V in perpendicular directions, collide and stick together. Their masses being in the ratio of 4 to 3, find the common velocity after impact.

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VI. PART III. FIRST PAPER.

[*Full marks will be given for about* two-thirds *of this paper.*]

1. Prove that if through any point P on an ellipse a line is drawn perpendicular to the major axis to meet the auxiliary circle in p, on the same side of the major axis as P, the tangents at P and p to the ellipse and circle meet in a point on the major axis.

Prove also that if q corresponds to Q in the same way as p to P, and if the tangents to the ellipse at P and Q meet in T and the tangents to the circle at p and q meet in t, then Tt is parallel to the minor axis.

2. Prove that the triangle formed by the two asymptotes of a hyperbola and any tangent is of constant area.

Prove that if LL' and MM' are two tangents which meet one asymptote in L and M, and the other in L' and M', then LM' is parallel to $L'M$.

3. Find the cosine of the angle between the two straight lines whose equations are

$$ax+by+c=0 \quad \text{and} \quad a'x+b'y+c'=0.$$

Form the equation of the two straight lines passing through the point of intersection of the lines $3x+4y+7=0$ and $4x-3y+1=0$ which make angles of 60° with the former line.

4. Find an expression for the area of a triangle in terms of the co-ordinates of its angular points.

Find the area of the triangle whose sides are $y=x$, $y=2x$, $x+y-6=0$.

5. Form the equation of a circle which passes through the origin, has the line $y=mx$ for a diameter, and touches the line $my+x=a$.

6. Find the equation of the normal to the parabola $y^2=4ax$ at any point.

Prove that, if any two chords PQ, PR at right angles to each other are drawn through the point $P(x', y')$ of the parabola, QR meets the normal at P in a point on the line $y+y'=0$.

7. Find the equation of the diameter of the ellipse $x^2/a^2+y^2/b^2=1$ which is conjugate to the diameter $y=mx$.

Prove that the equation of the locus of a point dividing in the ratio 2 : 1 the chords of the ellipse which make an angle θ with the major axis is

$$\left(\frac{\cos^2\theta}{a^2}+\frac{\sin^2\theta}{b^2}\right)\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}-1\right)+8\left(\frac{x\cos\theta}{a^2}+\frac{y\sin\theta}{b^2}\right)^2=0.$$

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8. Determine the conditions that a system of forces acting in one plane on a rigid body may keep it in equilibrium.

A rigid uniform rod is supported by two light strings attached to its extremities and to two fixed points at the same level. Prove that if the strings make equal angles with the vertical they must be equal in length.

9. Explain what is meant by the coefficient of friction between two bodies.

A rigid uniform rod is supported with its lower extremity resting against a rough plane of inclination α by a force applied to its upper extremity in the direction of the lines of greatest slope of the plane. Prove that, if the rod makes an angle $\alpha+\beta$ with the vertical and an angle $\beta(<\alpha)$ with the normal to the plane, the coefficient of friction cannot be less than

$$\tfrac{1}{2}\sec\alpha\sec\beta\sin(\alpha-\beta).$$

10. Show how to find the direction and magnitude of the relative velocity of two points which are moving with given velocities.

Two particles move along two perpendicular lines with uniform accelerations f and g, starting together from the point of intersection of the lines with velocities u and v. Prove that if $ug=vf$ their relative velocity at any time is directed along the line joining them.

11. A particle is projected from a given point in a given direction with a given velocity, and moves under gravity; determine the vertex of its path.

Prove that if a particle is just to clear three parallel vertical walls of heights b, c, b above the level of the point of projection and at distances a apart, the direction of projection must make with the horizontal an angle

$$\tan^{-1}\frac{2}{a}\sqrt{\{c(c-b)\}}.$$

12. A wheel is rolling uniformly on a level road and a smooth heavy bead is free to slide along one of the spokes. Find the least velocity of the wheel for which the bead will be always in contact with the rim of the wheel.

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VII. PART III. SECOND PAPER.

[*Full marks will be given for about* two-thirds *of this paper.*]

1. Define a parabola and deduce that the square of the ordinate of a point on the curve varies as the abscissa.

A is the vertex of a parabola, P any point on the curve, M a point on the axis such that MPA is a right angle. Show that if PN is perpendicular to AM then MN is equal to the latus rectum.

2. Prove that the sum of the focal distances of a variable point on an ellipse is constant.

Having given the foci and the length of the major axis of an ellipse, show how to find the points on it at which the line joining the foci subtends a given angle. If the eccentricity of the ellipse is $\frac{1}{2}$, within what limits must the angle lie?

3. Find the equation of the straight line through the points whose co-ordinates are $(3, -1)$, $(-\frac{1}{2}, 2)$.

The x co-ordinate of a point is 3, and it lies on the line joining the origin to the point whose co-ordinates are $(5, 7)$. Find its y co-ordinate.

4. Find the perpendicular distance of the point (x, y) from the line whose equation is $x \cos \alpha + y \sin \alpha = p$.

Find the locus of a point which moves so as always to be at a given distance of 10 units from the straight line $4x + 3y = 25$.

5. Find the co-ordinates of the centre, and the radius, of the circle whose equation is $x^2 + y^2 + 2y + 1 = 2x$.

Where are the intersections of the circles

$$x^2 + y^2 = 4x + 6y \quad \text{and} \quad x^2 + y^2 + 2x + 2y = 0?$$

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6. Find the equations of the tangents to the parabola $y^2 = 4x$ at the points whose x co-ordinates are each equal to 1.

Show that they are at right angles and intersect where the directrix cuts the axis.

7. Defining the ellipse as the locus of a point the sum of whose distances from two fixed points is constant, find its equation, taking the line joining the fixed points as axis of x, and the middle point of this line as origin.

If $(x_1, y_1)(x_2, y_2)$ are the co-ordinates of two points on the ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1,$$

show that

$$\frac{(x-x_1)(x-x_2)}{a^2}+\frac{(y-y_1)(y-y_2)}{b^2}=\frac{x^2}{a^2}+\frac{y^2}{b^2}-1$$

is the equation of the line joining them. Deduce the equation of the tangent at any point.

8. Give an example of (1) stable, (2) unstable, (3) neutral equilibrium.

A number of cubical blocks are piled one on the other so as to form a staircase, the breadth of each step being 2″, and the side of each block being 1 foot. How many can be piled before the whole begins to topple?

9. A board in the shape of an equilateral triangle is hung horizontally by three vertical strings attached to its corners. Show where to place a weight so that the tensions in the strings may be as 1 : 2 : 3. (Neglect the weight of the board.)

10. An elevator-cage is suspended by a rope passing over a smooth pulley and having a counterpoise equal to the weight of the cage at its other end. The rope between the weight and the pulley passes through the cage and can be handled by anyone within. A person of weight W steps inside, and by the friction of his hands on the rope reduces the downward acceleration to $\frac{1}{4}g$. Find the tension on the rope produced by the friction of his hands. Take the weight of the cage to be W, and that of the rope to be negligible.

11. A wedge has one face resting on a smooth table and is free to move. A smooth particle of half its mass is placed on the other face, one foot from the edge. How long will it take to reach the table, the angle of the wedge being 30°?

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12. In Atwood's machine where two masses m_1 and m_2 ($m_1 > m_2$) are connected by a light string passing over a smooth pulley, find the tension of the string during the motion.

If the pulley ascends with a uniform acceleration equal to $\frac{1}{3}g$, find the change in the tension.

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MATHEMATICAL EXAMINATION PAPERS

FOR ADMISSION INTO

Royal Military Academy, Woolwich,

NOVEMBER, 1899.

I. PART I. FIRST PAPER.

[*Great importance is attached to accuracy.*]

1. From $\frac{1}{5}-\frac{1}{7}+\frac{1}{9}$ take $\frac{1}{3}(\frac{1}{3}-\frac{1}{5}+\frac{1}{7})$, and simplify the fraction

$$\frac{\frac{1}{14}-\frac{1}{2}+\frac{78}{15}-\frac{703}{210}}{\frac{1}{20}+\frac{1}{60}-\frac{1}{28}}.$$

2. If one perch of wire weigh 24 lb. 12 oz. and one mile of the same wire cost £27. 10*s*. 0*d*., how much would 8 cwt. 2 qr. 8 lb. of the wire cost?

3. Find the present worth of £504. 15*s*. 0*d*. due 73 days hence at $4\frac{3}{4}$ per cent.

4. Extract the square root of 108241.

How many whole numbers between 100 and 100,000 are perfect cubes?

5. A certain length of pathway has to be constructed; it is found that three men can construct one-fifth all but one mile in two days, whilst 18 men can construct one mile more than two-fifths in one day. What is the length of the path?

6. Two numbers differ by 6; show that if 9 be added to their product the sum will be a square number. If the square root of this sum be 13, find the two numbers.

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7. Multiply $2x^2 - 19x + 35$ by $2x^2 - 13x + 15$, and divide the result by $4x^2 - 16x + 15$.

8. Solve the equations

$$\text{(i.)} \quad \frac{2x - 1}{5} - \frac{3x + 1}{16} = \frac{5}{2};$$

$$\text{(ii.)} \quad \frac{2x^2 - 1}{5} - \frac{3x + 1}{4} = \frac{9}{10}.$$

9. Solve the equations

$$\left.\begin{aligned} ax + by &= a^2 + 2ab - b^2 \\ bx + ay &= a^2 + b^2 \end{aligned}\right\}.$$

10. Having given the first term and the common difference of an Arithmetic Progression, find the expression for the sum of n terms of the series.

Find the sum of 25 terms of the series

$$1 + 2 + 3 + 4 + \ldots,$$

and find the value of n when the sum of n terms of the series

$$1 + 2 + 4 + 8 + \ldots$$

exceeds by 186 the sum of 25 terms of the preceding series.

11. Write down the expression for the number of permutations of n things taken r together, and hence deduce the expression for the number of combinations of n things taken r together.

Of 15 men, 10 can row and cannot steer, and five can steer and cannot row; find how many boats crews of eight rowers and a coxswain can be formed out of the 15 men.

12. Write down the expression for the coefficient of x^r in the expansion of $(1 + x)^n$, and find the greatest coefficient in the expansion of $(1 + x)^8$.

Also find the value of x when the fifth term in the expansion of $(1 + x)^{\frac{7}{2}}$ is equal to the number 70.

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II. PART I. SECOND PAPER.

N.B.—*In questions on Geometry ordinary abbreviations may be employed, but the method of proof must be geometrical. Proofs other than Euclid's must not violate Euclid's sequence of propositions. In the absence of special directions to Candidates, any of the propositions within the limits prescribed for Examination may be used in the solution of problems and riders.*

1. Define an *isosceles triangle, a circle,* and *parallel straight lines.*

Construct an isosceles triangle for which each of the equal sides shall be double the base.

2. Show that any two sides of a triangle are together greater than the third side.

Show that of triangles described on a given base, and having a given area, that which has the least perimeter it isosceles.

3. Divide a given straight line into two parts so that the rectangle contained by the whole and one part may be equal to the square on the other part.

If the line be divided so that nine times the rectangle contained by the whole and one part is equal to four times the square on the other part, find the point of division.

4. Prove that the angles contained by a tangent of a circle and a chord of the circle drawn from the point of contact of the tangent are respectively equal to the opposite angles subtended at the circle by the chord.

Describe two circles to touch two given circles, the point of contact with one of these given circles being given.

5. The radii of two intersecting circles are respectively 15 inches and 13 inches, and the common chord of the circles 24 inches long. What length of the line joining their centres lies within both circles?

6. Construct the centre of the circle inscribed in a given triangle.

Prove that, if ABC is the triangle, I the centre of the circle, D the middle point of BC, L the point where AI produced meets BC, and P the foot of the perpendicular from A on BC, then L lies between P and D, the sides AB and AC being unequal.

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7. State the relation between the areas of similar triangles and the lengths of their sides.

On the base BC of a triangle ABC as diameter a circle is described cutting the sides AB and AC in P and Q. Prove that, if the triangle APQ is half the triangle ABC, then PQ is the side of a square which can be inscribed in the circle.

8. In one side AB of a parallelogram $ABCD$ a point P is taken so that $3AP = 5PB$. Find the point Q in CD such that the area of the figure $APQD$ is half the area of the parallelogram. Compare the areas of the triangles PQC and PQD with the area of the parallelogram.

9. A frustum of a pyramid is contained between two parallel planes distant 9 feet apart, and the areas of the parallel faces are 5 square feet and 20 square feet. Determine the height of the pyramid and the volume of the frustum.

10. A square whose side is of length 1 foot revolves about one diagonal. Determine, to the nearest square and cubic inch respectively, the area of the surface, and the volume of the figure traced out. ($\pi = 3{\cdot}14159$.)

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III. PART I. THIRD PAPER.

1. Define the *sine* and *cosine* of any angle.

Show that the sine of an acute angle is greater or less than the cosine, according as the angle is greater or less than 45°.

2. Prove by geometrical constructions that for all values of A

$$\text{(i.)}\quad \sec^2 A = 1 + \tan^2 A\,;$$
$$\text{(ii.)}\quad \sin A = -\cos(90^\circ + A).$$

3. If D be the number of degrees in the smallest positive angle having its sine $= \frac{4}{5}$, find the values of

$$\cos(180 - D)^\circ,\quad \sin(180 + D)^\circ,\quad \tan(360 - D)^\circ.$$

Also write down a general formula including all angles whose

$$\text{sine} = +\tfrac{4}{5} \text{ and cosine} = -\tfrac{3}{5}.$$

4. Write down (without proof) formulae expressing $\sin(\theta + \phi)$ and $\cos(\theta + \phi)$ in terms of the sines and cosines of θ and ϕ, and thence express $\tan 2\theta$ in terms of $\tan\theta$.

Show that

$$\sin 2\theta = 1 - \cos 2\theta \cot\left(\frac{\pi}{4} + \theta\right).$$

5. If θ be the circular measure of an angle, prove that, as θ is indefinitely diminished, the ratios $\theta : \sin\theta$, $\theta : \tan\theta$, approach to the limit unity.

A man standing beside one milestone on a straight road, observes that the foot of the next milestone is on a level with his eyes, and that its height subtends an angle of 2′ 55″. Find the approximate height of that milestone. ($\pi = \frac{22}{7}$.)

6. Write down the values of sin 36° and cos 36° as given by your tables. Calculate the sum of the squares of these numbers to six decimal places, and explain why the result differs from unity.

$ABCDE$ is an equilateral and equiangular pentagon. Show that if the distance of A from B or E be 34 inches, its distance from C or D will be 55 inches nearly.

7. In any triangle prove that

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

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If BC be 25 inches, and CA be 30 inches, and if the angle ABC be twice the angle CAB, find the angles of the triangle ABC, and show that the length of the third side AB is 11 inches.

8. P, Q, R are three villages. P lies 7 miles to the north-east of Q, and Q lies $11\frac{1}{4}$ miles to the north-west of R. Find the distance and bearing of P from R.

9. Define the logarithm of a number to a given base, and show that the logarithm of a number to a base b may be obtained by dividing its logarithm to a base a by $\log_a b$.

Find the logarithm of 83 to base 19 as accurately as your tables permit.

If $x = \log_a bc$, $y = \log_b ca$, and $z = \log_c ab$, prove that

$$xyz = x + y + z + 2.$$

10. Point out the use of logarithms in facilitating numerical calculations, and employ the tables to obtain the values of

$$\text{(i.)}\quad \frac{327{\cdot}4 \times \sqrt[3]{0{\cdot}0006}}{\sqrt{62{\cdot}81}};$$

$$\text{(ii.)}\quad \frac{\sin 25^\circ \cos 37^\circ}{\tan 130^\circ}.$$

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IV. PART II. FIRST PAPER.

[*Full marks will be given for about* three-quarters *of this paper.*]

1. Investigate the connexion between the coefficients of the equation

$$ax^4 - bx^3 + cx^2 - dx + e = 0$$

and the simple symmetric functions of its roots.

Express (1) the sum of the squares, (2) the sum of the fourth powers of the roots of the equation in terms of the coefficients.

2. Find the general term of the series

$$(x+3)^{n-1} + (x+3)^{n-2}(x+2) + (x+3)^{n-3}(x+2)^2 + \ldots$$
$$+ (x+3)(x+2)^{n-2} + (x+2)^{n-1}$$

when it has been arranged in descending powers of x.

3. Prove that two circles cannot intersect in more than two points.

Draw a circle to pass through two given points, and to touch a given circle.

How must the positions of the points be limited?

4. A triangular plot of grass has sides a, b, c feet in length; it is bordered by a pathway m feet in breadth. Find the area of the pathway, and if the corners be rounded off by arcs of circles so that no part of the boundary is more than m feet from the grass, find by how much the area will be diminished.

5. Find an expression for $\cos(\alpha+\beta+\gamma)$ in terms of sines and cosines of α, β and γ.

Prove the identity

$$\cos\alpha\cos(\beta+\gamma) + \cos\beta\cos(\gamma+\alpha) + \cos\gamma\cos(\alpha+\beta)$$
$$= \cos(\alpha+\beta+\gamma) + 2\cos\alpha\cos\beta\cos\gamma.$$

6. A man has before him on a level plane, a conical hill of vertical angle 90°. Stationing himself at some distance from its foot he observes the angle of elevation α of an object which he knows to be half way up to the summit. Show that the part of the hill above the object subtends at his eye an angle

$$\tan^{-1}\frac{\tan\alpha(1-\tan\alpha)}{1+\tan\alpha(1+2\tan\alpha)}.$$

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7. Prove that there can be but one perpendicular to a plane from a point without the plane.

8. The base of a pyramid is an equilateral triangle, and the angles at the vertex are right angles. Show that the sum of the distances of the faces from any point in the base is constant.

If a side of the base be one foot, calculate the volume of the pyramid to the nearest cubic inch.

9. The tangents at points P, Q of a conic intersect in T. Prove that TP, TQ subtend equal or supplementary angles at a focus, and separate the cases.

10. Prove that all chords of a conic drawn through any point are cut harmonically by that point and its polar with respect to the conic.

A is the vertex, and AB the major axis of a conic. Let the tangent at any point P of the curve meet BA produced in T. Join PA, PB, and produce these lines to meet a straight line drawn through T perpendicular to AB in R and Q. Prove that $TR = TQ$.

11. P is a point on a conic whose vertex is A and focus S, and G is the foot of the normal through P. Prove that $SG = eSP$.

12. The ratio of the rectangles contained by the segments of any two intersecting chords of a conic is equal to that of the rectangles contained by the segments of any other two chords parallel to the former, each to each.

A, B, C, D are four points on a conic, which lie upon a circle. Show that the lines AB, CD are equally inclined to the axis, and that the same property is enjoyed by two other pairs of lines.

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V. PART II. SECOND PAPER.

[*Full marks may be obtained for about* three-quarters *of this paper. Gravitational acceleration may be taken equal to* 32 *feet per second.*]

1. State the triangle of forces.

Three forces act along the sides AB, BC, and AC of a triangle ABC, and are represented in magnitude by AB, BC, and twice AC respectively. Find the magnitude and line of action of their resultant.

2. If three coplanar forces are in equilibrium, prove that they must either be parallel or all meet in one point.

A uniform heavy rod rests with its extremities on two smooth inclined planes sloping towards each other at inclinations to the horizon of 30° and 60°. Find the inclination of the rod to the horizon in the position of equilibrium.

3. The uniform rod AB, 4 feet long, is hinged to a vertical wall at A, and supported in a horizontal position by the string CD attached to the rod at C and the wall at D, a point 4 feet above A. If the weight of the rod is 10 lb., and AC is 3 feet, find graphically the tension in the string, and the magnitude and line of action of the reaction at the hinge.

4. Find the magnitude and position of the resultant of parallel forces of 1, 2, 3, 4, 5 and 6 lbs. acting in vertical lines at distances 1 foot apart in the same vertical plane, the forces of 1, 3 and 5 lbs. acting upwards, and the forces of 2, 4 and 6 lbs. acting downwards. What is the distance of the resultant from the force of 1 lb.?

5. A wire is bent into the form of a triangle. Give a construction for its centre of mass.

If the sides of the triangle are 5, 12 and 13 inches in length, find the distances of its centre of mass from the two shorter sides.

6. Draw two different systems of weightless pulleys each of which gives a mechanical advantage of 8.

A number of pulleys of equal weight w are arranged in the system in which each pulley hangs by a separate string, and the top string passes over a fixed pulley to allow of the "power" being applied downwards. Prove that the weights of the pulleys may be counterpoised by attaching a weight w to the lowest pulley, and a weight w to the string intended to support the "power."

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7. Define *acceleration*, and explain how it is measured.

Find the initial velocity and the acceleration of a body which describes 30 centimetres in the first five seconds, and 50 centimetres in the next three seconds of its motion, the acceleration being uniform.

8. A body is projected vertically upwards with a velocity of 80 feet per second. Find and represent in a figure its positions after 1, 2, 3, 4 and 5 seconds respectively, and when at its greatest height.

If a second body is projected at the same instant with a velocity of 90 feet per second, how far apart will the two bodies be after 4 seconds?

9. State Newton's Second Law of Motion, and apply it to the following problems:

(i.) A body weighing 80 grains sliding down an incline of 30° from rest describes 5 feet in the first second. Find the retarding force up the plane.

(ii.) A body of weight W is acted on by gravity and by a force $\frac{1}{2}\sqrt{3}W$ acting in a direction inclined at 30° to the upward drawn vertical, find the direction in which the body begins to move.

10. A mass P on a smooth horizontal table is connected with unequal masses Q and R by strings passing over smooth pulleys at opposite edges of the table. Find the acceleration.

If $Q = 7$ lb. and $R = 2$ lb., the system starting from rest describes 5 feet in the first second. If $Q = 4$ lb. and $R = 1$ lb., the system describes 4 feet in the first second. From these data determine P and the acceleration of gravity.

11. A body is projected with velocity V in a direction inclined at an angle α to the horizon. Find the time of flight and range on a horizontal plane. Also prove that the former is twice the time taken to reach the greatest height, and the latter is twice the horizontal distance described when at the greatest height.

12. A particle is revolving with speed v in a circle of radius r. State and prove the formulae for the acceleration, pointing out in what direction this acceleration acts.

If the moon revolves about the earth in a circle whose radius is 240,000 miles, performing one revolution in 30 days, find its velocity and acceleration in foot-second units.

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VI. PART III. FIRST PAPER.

[*Full marks will be given for about* two-thirds *of this paper.*]

1. Prove that the principal ordinate of any point on the parabola is a mean proportional to the abscissa and the latus-rectum.

A diameter PM is drawn through the point P of a parabola so as to meet the tangent at the vertex A in M; MN is drawn at right angles to AP to meet the axis in N. Prove that AN is equal to the latus-rectum.

2. Given one focus and two points of an ellipse, find the locus of the other focus.

If the length of the major axis of the ellipse be given instead of one of the two points, show that the centre lies on a certain circle.

3. Construct the circle whose rectangular equation is

$$x^2+(y-c)^2=c^2.$$

Take a point P on the positive part of the axis of x; bisect OP, O being the origin, in Q and draw a straight line QAB passing through the centre of the circle and cutting it in the points A, B. Find the equation of the circle passing through the points P, A, B; find also its radius, and show that it touches the axis of x at P.

4. Describe the circle

$$x^2+y^2=c^2,$$

and draw the lines

$$(x\sqrt{3}+y-c)(-x\sqrt{3}+y-c)=0.$$

Find the equation of the line joining the feet of the perpendiculars upon these lines from any point (a, b) on the circle.

Prove that this line also passes through the foot of the perpendicular from the point (a, b) upon the line $2y+c=0$.

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5. Establish the equation of the normal to a parabola in the form

$$y - mx + 2am + am^3 = 0.$$

Find the co-ordinates of the point at which this straight line is the normal. If y_1 be the ordinate of this point and y_2 that of the middle point of the normal chord through the point, prove that

$$y_1 y_2 + 4a^2 = 0.$$

6. Chords of a parabola are drawn through a given point in its plane. Find the locus of their points of bisection, and discuss the nature, position, and dimensions of the locus for various positions of the given point.

7. Give a geometrical interpretation of the circumstance that the equation of a hyperbola results from the elimination of λ between the two equations

$$ay = \lambda b(x + a),$$

$$ay = \frac{b}{\lambda}(x - a).$$

8. A heavy particle hangs by a thread which is then drawn out of the vertical until it makes an angle with it. The particle is held in this position by a horizontally applied force, which after a time is suddenly withdrawn. If the tensions of the thread before the withdrawal and immediately afterwards be 4 poundsweight and 2 poundsweight respectively, find the weight of the particle.

9. Give an example of equilibrium of a body in which friction is one of the forces, but the limiting friction is not brought into play.

A uniform beam AB of weight w rests with one end A upon a rough horizontal plane and has its other end B attached to a string, which passes over a smooth pulley E and supports a weight P. Find the position of equilibrium for any value of the friction which does not exceed the limiting friction.

10. State Hooke's law in regard to the extension of an elastic cord.

A light extensible cord is attached to two points in a horizontal plane, its unstretched length being equal to the distance between the points. At its middle point is placed a heavy bead so that in the position of equilibrium the parts of the cord each make a very small angle, whose circular measure is α, with the horizontal. Show that if the extension of the cord be proportional to the tension and a force of P pounds be necessary to double the length of the cord, the weight of the bead is approximately $\alpha^2 P$ pounds.

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11. In the case of the path of a projectile in vacuo, show that, for a given velocity of projection, there are in general two directions of projection which will cause a given point to be struck. If the two directions coincide, prove that the angle of elevation is $\tan^{-1}\frac{2h}{a}$, where a is the horizontal distance of the point and h the height due to the initial velocity, and show further that the point is situated upon the parabola

$$x^2 = 4h(h - y),$$

the axes being through the point of projection, that of x horizontal and that of y vertical.

12. A uniform chain hangs by one end and sustains a load at the other. Show how to draw a diagram to represent the work done when the chain is wound up so as to raise the load.

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VII. PART III. SECOND PAPER.

[*Full marks will be given for about* two-thirds *of this paper.*]

1. Prove that the semi-latus rectum of any conic is a harmonic mean between the segments of any focal chord.

Prove also that the length of any focal chord of a parabola is equal to $4SP$, S being the focus, and P the extremity of the diameter which bisects the chord.

2. Give a construction for drawing two tangents to an ellipse from a given point.

Defining an *asymptote* of a hyperbola as a tangent whose point of contact is infinitely distant, show that the lines joining the centre of a hyperbola to the points where a directrix cuts the auxiliary circle are asymptotes according to this definition.

3. Interpret the constants in the equation of a straight line expressed

(i.) in the form $y = mx + b$;

(ii.) in the form $x = ny + a$.

If the axis of y is measured vertically upwards, prove that $y' - mx' - b$ represents the vertical height of the point (x', y') above the line whose equation is $y = mx + b$. Deduce the length of the perpendicular from the point (x', y') on the line $y = mx + b$.

4. Show that the equation

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + \frac{af^2 + bg^2 - 2fgh}{ab - h^2} = 0$$

represents two straight lines. If these are inclined to the axis of x at angles θ_1 and θ_2, express $\tan(\theta_2 - \theta_1)$ and $\tan(\theta_2 + \theta_1)$ in terms of a, h, b.

5. Find the co-ordinates of the middle point of the chord which the circle $x^2 + y^2 - 2x + 2y = 2$ cuts off on the line $y = x - 1$.

Find also the locus of the middle points of all chords of the circle which are parallel to the line $y = x - 1$.

6. Prove that the equations

$$y^2 = 4a(x + a) \quad \text{and} \quad y^2 = -4b(x - b)$$

represent a pair of parabolas having the same focus and axis. Find the co-ordinates of their points of intersection, and prove that at each of these points the tangents to the two parabolas are at right angles.

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7. Define the *eccentric angle* of any point on an ellipse. Find the equation of the chord joining the points on the ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1,$$

whose eccentric angles are θ, ϕ, and deduce the equation of the tangent at the point whose eccentric angle is θ.

Prove that the equation

$$\frac{x^2}{a^2\cos^2\alpha}+\frac{y^2}{b^2\sin^2\alpha}=1$$

represents, for different values of α, a series of ellipses all inscribed in a certain rhombus.

8. A uniform ladder of length l and weight w is being gradually tilted off the ground by a man who can reach up to a height h. If the ground be perfectly smooth where the lower end touches it, and if $l>2h$, prove that the ladder cannot be raised into a vertical position unless its lower end be weighted, and find the least weight that will suffice. If the ground be rough, show by a diagram that as soon as the inclination of the ladder to the vertical is less than the angle of friction the ladder may be supported at a point below its middle point, provided that a force of suitable magnitude and direction be applied to the point of support, and show how to determine this force when equilibrium is limiting, having given its point of application.

9. State Newton's Third Law of Motion, and show how it affords a means of comparing the masses of different bodies.

A body moving with velocity 5 feet per second overtakes another body moving with velocity 1 foot per second. The first body rebounds with velocity 1 foot per second, its direction of motion being reversed, while the second body has its velocity increased to 2 feet per second. Compare the masses of the bodies, and find the coefficient of restitution.

10. A stone is being whirled in a circle in a vertical plane at one end of a light string of which the other end is fixed. When the stone is at the highest point of the circle, the tension in the string just vanishes. Find the tension in the string when the stone is at the lowest point of the circle and when it is at the end of a horizontal diameter, expressing the result in terms of the weight of the stone. If the string be cut at the instant when the stone is ascending vertically, determine in terms of the radius of the circle the height to which the stone will rise.

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11. Prove that the work done in raising a number of weights through different heights is equal to the work which would be done in raising a single weight, equal to the sum of the weights, through a height equal to the height through which the centre of gravity of the weights is raised.

A lake whose superficial area is half a square mile has its surface at a height of 95 feet above the sea level. If the water is run off till the surface is lowered by ten feet, calculate the value of the work which can be done in driving machinery by this water in its descent to the sea, a cubic foot of water being assumed to weigh $62\frac{1}{2}$ lb., and work being valued at a halfpenny per horse-power per hour.

12. A number of particles are placed on a rectangular sheet of paper. Given the masses of the particles and their distances from two adjacent edges of the sheet, find the distances of their centre of gravity from these edges.

A series of cubes of the same material are piled one above the other on a horizontal plane. The lengths of the sides of the cubes, beginning with the lowest, are $2a$, $2ar$, $2ar^2$, $2ar^3$, and so on in geometrical progression. If there are n cubes in the pile, find the height of the centre of gravity of the whole system above the horizontal plane. If the number of cubes is infinite, r being less than unity, prove that the height of the centre of gravity of the pile is

$$a\frac{1+r^3}{1-r^2}.$$

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MATHEMATICAL EXAMINATION PAPERS

FOR ADMISSION INTO

Royal Military Academy, Woolwich,

JUNE, 1900.

OBLIGATORY EXAMINATION.

I. PART I. FIRST PAPER.

[*Great importance will be attached to accuracy.*]

1. Express as single decimals

(1) $0{\cdot}125 \times 0{\cdot}9152 \div 0{\cdot}0715$.

(2) $\left\{\frac{1}{13} \times 1\frac{5}{7} + \frac{2}{49} \times 1\frac{2}{5}\right\}$ of $\frac{2\frac{2}{5}}{36\frac{6}{7}}$.

2. What income will a man receive from investing £1000 in a $2\frac{1}{4}$ per cent. stock at $112\frac{1}{2}$?

If he sells out his stock at 105, what loss of capital will he sustain?

3. Find the true discount on £3570. 18*s.* 4*d.*, due 50 days hence, at $3\frac{3}{4}$ per cent. per annum.

4. A beam, having a square section, is 9 ft. long, and weighs $3\frac{1}{2}$ cwt. A cubic foot of the substance of the beam weighs 32 lbs. What is the thickness of the beam?

5. A dishonest tradesman marks his goods at an advance of 5 per cent. on the cost price, but uses a fraudulent balance, whose beam is horizontal when the weight in one scale is one-fifteenth more than the weight in the other. What is his actual gain per cent.?

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6. A man walks a third as far again as a boy in a given time. His stride is 5 centimetres longer than the boy's, and he takes 125 strides per minute, while the boy takes only 100. At what rate in kilometres per hour does each walk?

7. Multiply $7x^2-23x+6$ by $6x^2-35x+11$, and divide the result by $3x^2-10x+3$.

8. Solve the equations

$$\text{(i.)}\ \frac{x-5}{2}-\frac{2x-11}{6}=\frac{11x-7}{140};\quad \text{(ii.)}\ 9x^2+17x-2=0.$$

9. Solve the equations

$$c^2x+2a^2y=(c+a)(cx+2ay)=(c-a)^2.$$

10. Having given the first term and the constant ratio of a geometric progression, find the expression for the sum of the first n terms.

Find the sum of ten terms of the geometric series

$$3+6+12+24+\ldots$$

What is meant by saying that the sum to infinity of the series

$$1+\tfrac{1}{2}+\tfrac{1}{4}+\tfrac{1}{8}+\ldots$$

is 2?

11. Find the number of combinations of n things, r at a time, not assuming the formula for the number of permutations.

In how many ways could a party of five scouts be selected from 20 available men? In how many ways could the twenty men be formed into four parties of five scouts, to proceed in different directions?

12. Prove the binomial expansion, n being a positive integer.

If $$(1+x)^n=1+n_1x+n_2x^2+n_3x^3+\ldots,$$
where $$n_r=n(n-1)\ldots(n-r+1)\div r!,$$
find the value, when $n=9$, of $1-n_2+n_4-n_6+n_8$.

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II. PART I. SECOND PAPER.

N.B.—*In questions on Geometry ordinary abbreviations may be employed, but the method of proof must be geometrical. Proofs other than Euclid's must not violate Euclid's sequence of propositions. In the absence of special directions to Candidates, any of the propositions within the limits prescribed for Examination may be used in the solution of problems and riders.*

1. Prove that two triangles standing on the same base and between the same parallels are equal in area.

Hence show that if two triangles have two sides of the one equal to two sides of the other and the contained angles supplemental, they are equal in area.

2. Any point, P, is taken on a diagonal of a parallelogram, and through the point are drawn two parallels to the sides; prove that the complements of the parallelograms about the diagonal are equal in area.

Ascertain for what position of P on the diagonal the area of each of these complements is greatest.

3. Show how to find the centre of a given circle.

Being given an arc of a circle, show how to describe the whole circle.

4. Prove that the angle subtended at the centre of a circle by any arc is double the angle subtended by the arc at any point on the circumference.

From this show that the sum of a pair of opposite angles of a quadrilateral in a circle is two right angles.

5. Show how to divide a given right line into three equal parts.

From the process employed (or otherwise) show that the bisectors of any two sides of a triangle drawn from the opposite vertices trisect each other.

6. Inscribe a circle in a given triangle.

ABC is a right-angled triangle, A being the right angle. Prove that the hypotenuse BC is equal to the difference between the radius of the inscribed circle of the triangle and the radius of the circle which touches BC and the other two sides produced.

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7. Prove that the sides about the equal angles of triangles which are equiangular to one another are proportionate, and that those which are opposite to the equal angles are proportionals.

Find the point P in the side AC of a triangle ABC such that the rectangle contained by AP and AC is equal to the square on AB.

8. AB is a chord of a circle, centre C, equal in length to the radius of the circle. If the length is eighty-five inches, find, to the nearest inch, the length of the arc AB.

$$[\pi = 3{\cdot}1416.]$$

9. The cross-section of a wedge, in the form of a right triangular prism, is an equilateral triangle, the side of which is eleven inches; if the length of the wedge is thirty-seven inches, find, to the nearest integer, the number of square inches in the total surface of the wedge.

10. A railway cutting 8 metres deep has to be made, with one side vertical and the other inclined at 30 degrees to the vertical; the bottom is to be 9·4 metres broad. How many cubic metres (to the nearest integer) of earth must be removed per kilometre?

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III. PART I. THIRD PAPER.

1. Explain the sexagesimal measure of angles.

The earth revolves once in 24 hours; express in sexagesimal measure the angles through which it revolves (1) in one hour, (2) in one minute of time, (3) in one second of time.

2. Define the *cosine* of an angle, explaining how its algebraic sign is determined. In which quadrants is the cosine positive?

Construct an angle whose cosine is $-\frac{5}{13}$, and find its remaining trigonometric ratios.

3. State and prove the values of the sine, cosine, and tangent of 30°.

A man 6 feet high observes the elevation of a tower to be 30° when he is standing 80 feet from the base of the tower; find (to the nearest foot) the height of the tower.

4. Prove that $\sin^2\theta + \tan^2\theta = \sec^2\theta - \cos^2\theta$, and solve the equation

$$\sec\theta + \tan\theta = \sqrt{3}.$$

5. If
$$x = X\frac{\sin(\theta-\alpha)}{\sin\theta} + Y\frac{\sin(\theta-\phi-\alpha)}{\sin\theta},$$
and
$$y = X\frac{\sin\alpha}{\sin\theta} + Y\frac{\sin(\phi+\alpha)}{\sin\theta},$$
express $x^2 + 2xy\cos\theta + y^2$ in terms of X, Y, and $\cos\phi$.

6. Assuming expressions for the expansions of $\sin(A+B)$ and $\cos(A+B)$ establish the formulae

$$\sin 2A = 2\sin A\cos A$$
$$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}.$$

7. One side of a right-angled triangle is 6·432 feet long, and the angle opposite to it is 37° 27′. Find (i.) the area of the triangle; (ii.) the length of the perpendicular from the right angle on the hypotenuse.

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8. In any triangle show that

$$4bc\cos^2\tfrac{1}{2}A = (a+b+c)(-a+b+c).$$

If $a = 17$, $b = 18$, $c = 19$, calculate A as accurately as your tables permit.

9. Prove that the logarithm of a product is the sum of the logarithms of its factors, the base remaining the same.

If $e = 2{\cdot}71828$, employ the tables to find $\log_{10}2$, $\log_e 10$, and $\log_e 2$.

10. (i.) Solve the equation

$$2x^2 = 16^{x-1}.$$

(ii.) Find the number of digits in 19^{33}.

(iii.) Find the number of zeros following the decimal place in the value of $(\frac{1}{19})^{33}$.

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IV. PART II. FIRST PAPER.

[*Full marks will be given for about* three-quarters *of this paper.*]

1. Obtain the common difference, the nth term and the sum of n terms of the arithmetic progression whose first and second terms are u and v respectively.

Show that this sum may be written in the form $nu - \frac{n(n-1)}{1 \cdot 2}\frac{u^2}{s}$ where (if $\frac{v}{u}$ is a proper fraction) s is the "sum to infinity" of the geometric progression $u, v, \ldots$

2. Write down the rth term of the expansion of $(1+x)^n$ in a series of ascending powers of x. Hence obtain, and reduce to its simplest form, the rth term (u_r) in the expansion of $(1-qx)^{-p/q}$.

If p, q are positive integers, p being less than q, and if qx is a positive proper fraction, show that the ratio $u_{r+1} : u_r$ is positive and less than qx. Hence prove that the sum of the terms which follow the rth term ("the remainder after r terms") is less than $\frac{qx}{1-qx}u_r$.

3. Prove that the straight line joining the middle points of two sides of a triangle is parallel to, and is equal to half the length of the third side.

If $ABCD$ be any four-sided figure the three straight lines joining the middle points of AB and CD, of AC and BD, of AD and BC respectively, meet in one point which bisects each of the joining lines.

4. Construct an isosceles triangle having each of the angles at the base double the vertical angle.

A is the vertex and BD the base of the isoceles triangle; C is the point on AB used in the construction; M is the middle point of BD, and AM intersects CD in N. Show that the ratio $AM : NM$ is equal to the ratio of the perimeter of ABD to its base.

5. A, B, C are three angles whose sum is $2S$, and θ is an angle defined by the equation

$$\cos\theta = \frac{\cos A + \cos B \cos C}{\sin B \sin C};$$

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prove that

$$\cos \tfrac{1}{2}\theta = \sqrt{\frac{\cos(S-B)\cos(S-C)}{\sin B \sin C}}.$$

6. The lengths of the sides of a triangle are 375 links, 452 links, and 547 links. Find the length of the perpendicular upon the shortest side from the opposite corner, and the radius of the inscribed circle.

7. If a straight line be at right angles to two other straight lines which cut one another then it is at right angles to the plane containing these lines.

8. A solid cube (edge, 10 inches long) is perforated so that a square pyramid just fits into the hole. The pyramid has a base equal to a face of the cube and a height of 20 inches, and when it is fitted into the hole its base lies flush with the surface of the cube, and its vertex projects beyond the face opposite to the base. What is the area of the whole surface (inside and outside) of the perforated cube?

9. State the focus-directrix definition of a conic section. Show how points upon the conic may be found directly from the definition.

An arc, AB, of a circle is bisected by a diameter of the circle. If this diameter be taken as directrix and the point A be taken as focus, find the eccentricity so that the conic may intersect the circular arc in one of the two points that divide it into three equal parts.

10. Given a focus, the corresponding directrix and one point, P, of a conic construct the tangent at P.

Prove the proposition you use for this purpose.

11. PT is a straight line touching a conic at P. TK is at right angles to the focal distance SKP and TH is at right angles to the directrix XH. Prove that $SK : TH$ is equal to the eccentricity.

12. Prove that the semi-latus rectum is the harmonic mean between the two segments of any focal chord.

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V. PART II. SECOND PAPER.

[*Full marks will be given for about* three-quarters *of this paper.*]

1. Show how to resolve a given force into two components, one of which is given in magnitude, direction and sense.

A force R is resolved into two components, one of them being in a line making an angle θ with the line of action of R; find the least possible magnitude of the other.

2. A is a point 4 feet to the left of and 1·2 feet vertically above B. A string, attached to the point A, passes over a smooth fixed peg B, and supports a weight of 19 lbs.; at a point C of the string a second string is knotted to the first and pulled downwards with a tension P. Determine the magnitude of P. ($AC = 3{\cdot}2$ feet, $BC = 2{\cdot}7$ feet.)

3. A light rod AB, length 3·2 feet, can turn in a vertical plane about a hinge at A, and supports a weight of 10 lbs. at D, its middle point, and the system is supported in a horizontal position by a light rod CB, C being fixed 2·4 feet vertically under A; find graphically the thrust in the rod CB.

4. Prove that, if the lines of action of two forces intersect, the sum of their moments about any point in their plane is equal to the moment of their resultant about the same point.

A uniform circular disc of weight W is supported in a vertical plane on two smooth pegs, which are so placed that the line joining them subtends a right angle at the centre of the disc, and makes an angle α ($<45°$) with the horizontal. Determine the pressure on each peg.

5. Find the magnitude and line of action of the resultant of two parallel forces acting in the same sense on a rigid body.

The bar of a common steelyard is 2 feet long, weighs 3 lbs., and is suspended at a point distant 2 inches from one end, and its centre of gravity is distant 3 inches from the same end. Find the greatest weight that can be weighed with this steelyard, the movable weight being 5 lbs.

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6. Three pieces of wire of equal weights and unequal lengths are joined together so as to form a triangle. Find its centre of gravity.

Three pieces of wire of equal lengths, weighing 1, 2, and 3 grains respectively, are joined together so as to form a triangle which is hung up by the corner opposite to the heaviest side. Find the depth of the centre of gravity below the corner.

7. Prove the formula for uniformly accelerated motion $s = ut + \frac{1}{2}ft^2$.

A particle moves from a point A in a straight line with uniform velocity v; and a second particle starts from rest at A at the same instant, and moves in the same direction along the same line with uniform acceleration f. Find the greatest distance between the particles before the second overtakes the first.

8. The velocities of two moving points being given, determine their relative velocity.

Two particles move with constant velocities along two straight lines which cut at right angles; prove that, if the second moves through the point of intersection with velocity v at an instant when the first is distant a from that point, the angular velocity of the line joining them when the distance between them is r is av/r^2.

9. A body of mass m slides down a plane of inclination α to the horizontal under gravity and a constant resistance F, acting up the line of slope; find its velocity after descending a height h.

If the body is projected up the plane with this velocity, and moves under gravity and the resistance F, acting down the line of slope; find how far it ascends before coming to rest.

10. Two bodies whose masses are m and m' are connected by a cord passing over a smooth pulley. The pulley is at a height $2h$ above a table, on which is another body of mass $m - m'$, vertically under m', and attached to m' by another cord of length h. The system is let go from a position in which m' is at a height $\frac{1}{2}h$ above the table, and the table does not interfere with the motion of m. Find the velocity of m' just before the second cord becomes tight.

Find also the velocity of m just after the second cord becomes tight.

11. A particle is projected from a given point with a given velocity. Find the range on an inclined plane through the point of projection.

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Prove that, if the angle of elevation at starting is such as to make the range as great as possible, the velocity of the particle when it strikes the plane will be as small as possible, the further extremity of the range being at a higher level than the point of projection.

12. A particle of mass m is describing a circle of radius r with uniform speed v. State (without proof) the magnitude and direction of the force acting upon it.

To a string, of which one end is fixed, are attached two particles of equal masses at the further end and the middle point; and the two particles describe circles uniformly, with the two portions of the string always in a straight line. Prove that the tensions of the two portions are in the ratio 2 : 3.

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VOLUNTARY MATHEMATICS.

VI. PART III. FIRST PAPER.

[*Full marks will be given for about* two-thirds *of this paper.*]

1. The tangents at two points P, Q on a parabola, whose focus is S, meet at T; show that the angles TSP, TSQ are equal.

If the angle PTQ be one-third of two right angles and the finite line ST cut the parabola at R then the tangents at P, Q, R form an equilateral triangle.

2. Prove that the feet Y, Y' of the perpendiculars SY, $S'Y'$ drawn from the foci S, S' of an ellipse to the tangent at any point P lie on the major auxiliary circle.

From Y another tangent YQ is drawn to the ellipse; show that SQ is parallel to $S'P$.

3. Prove that the area of the triangle formed by the origin and the two points whose rectangular co-ordinates are (x_1, y_1), (x_2, y_2) is $\frac{1}{2}(x_1y_2 - x_2y_1)$.

Find the length of the perpendicular drawn from the origin to the line joining the points whose rectangular co-ordinates are (4, 1), (7, 5).

4. Show that each of the circles

$$x^2+y^2-2x\sqrt{3}+2y=0 \text{ and } x^2+y^2+\frac{2}{\sqrt{3}}x-2y=0$$

circumscribes an equilateral triangle which has one side lying along an axis of co-ordinates.

Find the co-ordinates of the second point of intersection of these circles.

5. Show that the parabolæ whose equations in rectangular co-ordinates are $y^2-ax=0$, $x^2-by=0$ have only one common chord which cuts the curves in real points.

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If P and P' be the points on $y^2 - ax = 0$ and $x^2 - by = 0$ the tangents at which are parallel to this common chord, and S, S' be the foci of the parabolæ, then SP is perpendicular to $S'P'$.

6. If S and S' are the foci, SL the semi-latus rectum of an ellipse, and LS' produced cuts the ellipse at X, show that the length of the ordinate of X is $\frac{(1-e^2)^2}{1+3e^2}a$ where $2a$ is the length of the major axis and e is the eccentricity of the ellipse.

7. Find the equation of the tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point (x', y').

If y_1, y_2 be the ordinates of the points in which this tangent cuts the major auxiliary circle show that $\frac{1}{y_1} + \frac{1}{y_2} = \frac{2}{y'}$.

8. A smooth pencil, whose cross-section is a circle of radius r, is laid upon a flat and smooth ruler whose breadth is $2c$, so that its axis is parallel to the edge of the ruler; the two are now bound together by a string which, passing under the ruler and over the pencil, lies in a plane perpendicular to the axis of the pencil; show that the pressure between the pencil and the ruler is $\frac{4rc}{r^2+c^2}T$ where T is the tension of the string.

9. Enunciate the laws of statical friction.

A pair of step-ladders stands on a rough horizontal plane with each of its ladders on the point of slipping; show that the feet of the ladders are at a distance $\frac{4\mu l}{\sqrt{1+4\mu^2}}$ apart, where μ is the coefficient of friction, and l is the length of each ladder. (The centre of gravity of each of the ladders is to be supposed to be half way up the ladder.)

10. Distinguish between stable, neutral, and unstable equilibrium.

A uniform beam AB, whose weight is W, is moveable in a vertical plane about a hinge at A; to the end B, one end of a string which carries a weight W at its other end is attached; the string is passed over a pulley at a height h ($h > AB$) vertically over A; determine whether the beam is in stable or unstable equilibrium when it is in a vertical position above A.

11. Explain the meaning of the terms Work, Kinetic Energy.

Find the greatest velocity (in miles per hour) at which an engine of 192 horse-power can pull a train of 150 tons mass along the level, the

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frictional and other resistances to the motion being 16 pounds weight per ton. (A horse-power is 550 foot-pounds per second.)

12. State the laws which determine the velocities, after impact, of two smooth elastic spheres whose velocities before impact are known.

A smooth billiard ball strikes another (of equal size) which is at rest, and the direction of the centre of the former makes, just before impact, an angle of 30° with the line joining the centres of the two balls; find the tangent of the angle through which its direction of motion is deflected by the impact, the coefficient of restitution of the balls being 0·4, and the motion being supposed to take place upon a smooth billiard table.

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VII. PART III. SECOND PAPER.

[*Full marks will be given for about* two-thirds *of this paper. The acceleration due to gravity may be taken to be* 32 *foot-second units.*]

1. Having given a focus, and one tangent of a parabola, and the length of the latus rectum, find the point of contact of the given tangent.

2. Prove that any tangent of a hyperbola cuts off from the asymptotes a triangle of constant area.

If the tangent at P meets an asymptote in L, and the perpendicular from the centre C on the tangent PL is equal to the perpendicular from L on CP, the asymptotes are at right angles.

3. The equation of a straight line being expressed in the form

$$\frac{x-a}{\cos\theta} = \frac{y-b}{\sin\theta}$$

interpret the geometrical meaning of either member of the equation.

Straight lines parallel to $y = kx$ are drawn to cut the two given lines $y = mx$ and $y = nx$; find the equation of the locus of the middle points of the parallel intercepts.

4. Find the equation of the two straight lines drawn through the point (1, 1) which are parallel respectively to the straight lines given by the equation $x^2 + 5xy + 2y^2 = 0$.

Find also the equations of the two diagonals of the parallelogram formed by the four lines.

5. Form the equation of a circle of radius a touching both the axes of co-ordinates, and lying in the quadrant in which both co-ordinates are positive.

Find the angles which a chord PQ passing through the origin O must make with the axes in order that OQ may be bisected at P.

6. Find the equation of the tangent to the parabola $y^2 = 4ax$ at the point $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$.

Prove that, if two tangents are drawn from a point on the line $x = -4a$, their chord of contact subtends a right angle at the vertex.

7. Form the equation of the diameter of the ellipse $x^2/a^2 + y^2/b^2 = 1$ which is conjugate to the diameter $y = x \tan\theta$.

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Chords AP, AQ parallel to the two diameters are drawn through the vertex (a, o). Prove that PQ makes with the axis of x an angle

$$\tan^{-1}\frac{2\tan\theta}{1-\frac{a^2}{b^2}\tan^2\theta}.$$

8. A rigid body is under the action of any number of forces in one plane. State the conditions of equilibrium.

A vertical post is held upright in contact with the ground by two cords, attached to pegs on the ground at equal distances c from the foot of the post, and attached to the post at heights a and b above the ground; the cords being on either side of the post in a vertical plane. Prove that the tangent of the angle which the reaction of the ground makes with the vertical cannot exceed $\frac{1}{2}c(a-b)/ab$.

9. Two particles of given masses are moving in a plane with given velocities in given directions. Determine the velocity of their centre of mass.

If the masses of the particles are m and m', and the velocity of m relative to m' is of magnitude v and makes an angle α with the line drawn from m' to m, determine the magnitude and direction of the velocity of m relative to the centre of mass.

10. A body which weighs 10 lbs. is attached to a cord passing over a fixed smooth pulley, and the cord is drawn over the pulley at a uniform rate of 3 ft. per sec. What is the magnitude of the tension of the cord?

If the pulley is rigidly attached to a moving platform, which is ascending with an acceleration of 4 foot-second units, and the cord is drawn over the pulley as before, what change is made in the tension?

11. Two particles of equal masses are attached to the ends of a light string of length l; one of them, A, is placed on a smooth table and the other, B, hangs just over the edge, the string being just tight. Find the magnitudes and directions of the velocities of the two particles (1) just before A leaves the table, (2) just after.

12. A particle is projected from a given point with a given velocity in a given vertical plane; find the envelope of the possible paths.

A shot is fired from a gun at the top of a cliff of height h with a velocity u ft. per sec. Prove that, if the range measured from the foot of the cliff is as great as possible, the elevation α is given by the equation

$$\cos 2\alpha = gh/(gh+u^2).$$

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MATHEMATICAL EXAMINATION PAPERS

FOR ADMISSION INTO

Royal Military Academy, Woolwich,

NOVEMBER, 1900.

OBLIGATORY EXAMINATION.

I. PART I. FIRST PAPER.

[*Great importance is attached to accuracy.*]

1. Express as a fraction in its lowest terms

$$\{\tfrac{6}{11} \text{ of } 17\tfrac{2}{7} + \tfrac{19}{55} \text{ of } 116\tfrac{4}{19}\} \times \{\tfrac{11}{1700} \text{ of } 144\tfrac{1}{8}\}.$$

2. Calculate to five places of decimals (so that the error is less than 0·000005) the value of the expression

$$\frac{14{\cdot}52}{83{\cdot}1} + \frac{1{\cdot}575}{13{\cdot}85}.$$

3. A rectangular field is 330 yards in length and 188 yards in breadth; find the number of acres in the field, and the money which would be obtained by selling half the field for £17. 4*s*. 6*d*. an acre, and the other half for £21. 15*s*. 6*d*. an acre.

4. A person, having four thousand pounds to invest, places £2000 in $2\frac{3}{4}$ per cent. consols at $98\frac{1}{2}$, and £2000 in India three per cents. at $98\frac{5}{8}$; find, to the nearest penny, the income thereby obtained.

5. A railway train, 73 metres in length, is travelling at the rate of 60 kilometres an hour on the up line, and another train, 102 metres in length, is travelling in the opposite direction, on the down line, at the rate of 40 kilometres an hour; find the time occupied by the trains in passing each other.

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If the trains were travelling in the same direction, what would be the time occupied by the faster train in passing the slower train?

6. A clockmaker was asked how many clocks he had, and replied: I have more than 500 but less than 1000; if I count them four by four, or five by five, or six by six, there remains one over, if I count them by sevens none remain over. How many clocks had he?

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7. Show that

$$(ax+by+cz)^2+bc(y-z)^2+ca(z-x)^2+ab(x-y)^2$$
$$=(a+b+c)(ax^2+by^2+cz^2).$$

8. Determine c so that the equation in x

$$\frac{x-3}{x-4}+\frac{x-4}{x-3}=\frac{2x-c}{x-5}$$

has zero for a root. With this value of c solve the equation completely.

9. Solve the simultaneous equations

$$x(b-c)+by-c=0, \quad y(c-a)-ax+c=0.$$

Show that the values which satisfy the above equations will also satisfy the equation $ax-by+a-b=0$.

10. Prove the Index Law, namely that $a^m \times a^n = a^{m+n}$, when m and n are positive integers.

If the Index Law be assumed to hold good for all values of the indices, find what meanings must be assigned to a^0, a^{-m}, and $a^{\frac{m}{n}}$.

Multiply $a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}-2b^{\frac{2}{3}}$ by $2a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}-b^{\frac{2}{3}}$, and divide the product by $2a^{\frac{2}{3}}-3a^{\frac{1}{3}}b^{\frac{1}{3}}-2b^{\frac{2}{3}}$.

11. Find the arithmetic mean of x and y.

If a, b, c, are in arithmetical progression, show that $2a-3b$, $2b-3c$, and $3b-4c$ will also be in arithmetical progression.

Show that $3(1+3+5+\ldots+99)=101+103+105+\ldots+199$.

12. Write down expansions of $\frac{1}{\sqrt{1+x^2}}$, one of them valid when $x<1$, the other when $x>1$.

Use one of the expansions to find its value to the nearest millionth when $x=19$.

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II. PART I. SECOND PAPER.

N.B.—*In questions on Geometry ordinary abbreviations may be employed, but the method of proof must be geometrical. Proofs other than Euclid's must not violate Euclid's sequence of propositions. In the absence of special directions to Candidates, any of the propositions within the limits prescribed for Examination may be used in the solution of deductions.*

1. (i.) Define a *parallelogram.*

(ii.) If, of the sides of a quadrilateral, one pair is parallel and the other pair equal, prove that any angle is either equal or supplementary to the opposite angle.

2. (i.) Prove that the exterior angle of a triangle equals the sum of the two interior and opposite angles.

(ii.) In a certain quadrilateral the sum of two internal adjacent angles is 200°. Find, in degrees, the size of the obtuse angle contained by the bisectors of the two remaining internal angles of the quadrilateral.

3. (i.) Show, with accompanying proof, how to construct a square equal to a given rectilinear figure.

(ii.) By means of a figure carefully drawn to scale, *apply the construction in* (i.) to find, as accurately as you can, the lengths of the sides of a rectangle whose area is 9 sq. in. and whose perimeter (the sum of the sides) is 14 inches. A proof is not required.

4. Prove that the sum of two opposite angles of a quadrilateral inscribed in a circle equals two right angles.

5. (i.) Give, without the proof, the construction for drawing a tangent to a circle from an external point.

(ii.) P, C and L denote respectively a given point, circle and unlimited straight line, P and C lying on the same side of L. Find, with proof, a point Q in L such that PQ and a tangent from Q to C (not in the same line with PQ) make equal angles with L. Only one solution is required.

6. Show how to circumscribe a circle about a given triangle ABC.

If O is the centre of this circle, and the lines AO, BO, CO produced meet the circumference in A', B', C', show that the triangle $A'B'C'$ is in every respect equal to ABC.

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7. Show how to find a mean proportional between two given right lines.

O is a point outside a given circle; through O is drawn any right line meeting the circle in A and B; state (without proof) why the mean proportional between OA and OB is the same for all lines drawn through O, and exhibit this mean proportional.

8. Prove that the area of a trapezium is equal to half the sum of the parallel sides multiplied by the perpendicular distance between them.

The parallel sides of a trapezium are 9 and 30 feet long, and the other sides are 17 and 10 feet long; find its area.

9. On the base of a hemisphere of radius r is constructed a right cone whose volume is equal to that of the hemisphere; what is the height of the cone?

10. A spherical shell 1 foot in external diameter and 2 inches thick is made of metal having a mass of 504 pounds per cubic foot; find (to the nearest pound) the mass of metal in the shell. (The circumference of a circle is 3·14159 times the diameter.)

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III. PART I. THIRD PAPER.

1. State the most approximate value you know for the numerical quantity π, and compare it with the value of the fraction $\frac{355}{113}$ expressed as a decimal.

What is meant by an angle π, and why?

Calculate the value of a radian expressed in seconds.

2. Express all the other trigonometric functions of A in terms of $\sin A$. Why must the double sign $\pm$ be prefixed to all the square roots occurring in your expressions?

Write down a general formula for all angles whose sine is equal to $\frac{1}{2}$ expressed *either* in degrees *or* in radians.

3. Simplify

$$(\sec\theta - \cos\theta)(\operatorname{cosec}\theta - \sin\theta)(\tan\theta + \cot\theta)$$

and find a positive angle θ less than $180°$ satisfying the equation

$$1 - \cos\theta = \sqrt{3}\sin\theta.$$

4. Write down and prove the formula giving $\tan(A+B)$ in terms of $\tan A$ and $\tan B$.

The perpendicular from the vertex of a triangle on the base is 6 inches long, and it divides the base into segments which are 2 and 3 inches long respectively. Find the tangent of the vertical angle of the triangle.

5. An observer wishing to determine the length of an object in the horizontal plane through his eye, finds that the object subtends the angle α at his eye when he is in a certain position A. He then finds two other positions B and C where the object subtends the same angle α. Express the length of the object in terms of the sides of the triangle ABC and of the angle α.

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6. What is meant by the supplement of an angle? What is the supplement of an angle of $330°$? Draw a figure showing the angle and its supplement, and prove that the sines of these two angles are algebraically equal.

Find $\operatorname{cosec} 630°$.

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7. Prove the formula

$$a^2 = (b + c - 2\cos\frac{A}{2}\sqrt{bc})(b + c + 2\cos\frac{A}{2}\sqrt{bc}).$$

Apply it to find the side a of a triangle when $b = 132{\cdot}5$ feet, $c = 97{\cdot}32$ feet, $A = 37^\circ\ 46'$, as accurately as the tables permit.

8. Find an expression for the difference in area of the two triangles which can be drawn having two given sides a, c and a given angle A opposite the smaller side a.

9. Explain the meaning of $\log_a m$.

Prove that $\log_a m \times \log_m a = 1$.

Without using the tables find the characteristics of (1) $\log_7 15914$, (2) $\log_8 0{\cdot}00187$.

10. Throw the expression

$$\frac{5\sin\theta + 3{\cdot}584\cos\theta}{5\sin\theta - 3{\cdot}584\cos\theta}$$

into a form suitable to logarithmic calculation when different values of θ are introduced, and use your form to evaluate the expression when $\theta = 71^\circ\ 59'$ correct to three places of decimals.

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IV. PART II. FIRST PAPER.

[*Full marks will be given for about* three-quarters *of this paper.*]

1. (i.) Simplify

$$\frac{x^2y^2(x^2-y^2)+y^2z^2(y^2-z^2)+z^2x^2(z^2-x^2)}{xy(x-y)+yz(y-z)-zx(z-x)}.$$

(ii.) If the sum of a, b, and c be p, show that the sum of $a(b+c)$, $b(c+a)$, and $c(a+b)$ cannot be greater than p^2.

2. Trace the graph of the function x^3-4x+1. (The *graph* is the locus of a point whose distance from one straight line is x, and whose distance from a perpendicular straight line is the corresponding value of x^3-4x+1.)

From your diagram show that the given function vanishes for three values of x, and find between what pair of integers each of these values lies.

3. ABC is a triangle; BA, CA are produced through A to points D, E, so that ED and BC are parallel. Prove that the circles ABC, ADE touch one another.

4. Prove that the volume of a tetrahedron is one-third that of the prism on the same base and of the same altitude.

In a regular tetrahedron prove that the perpendicular from a summit on the opposite face is three times the perpendicular from the centre of gravity of a face upon any other face.

5. Express the tangent of an angle in terms of the cosine, explaining the ambiguity of sign.

Having given $\tan\theta=\sqrt{15}+2\sqrt{3}$, $\cos\alpha=\frac{1}{4}$, and that α is an acute angle, show that $\tan(\pi-\alpha-\theta)=\sqrt{15}-2\sqrt{3}$.

6. Investigate some expression for the radius of the circle circumscribing a triangle, and write down any other expressions you know.

A man is on the perimeter of a circular field, and wishing to know its diameter, he selects two points in the boundary a furlong apart, which at a third point, also in the boundary, subtend an angle of $164^\circ\ 43'$. Find the diameter to the nearest foot.

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7. Two intersecting straight lines are respectively parallel to two other intersecting straight lines. Show that the plane containing the first pair is parallel to that determined by the second pair, or else coincides with it.

8. Define a trihedral angle, and show that the sum of any two of the plane angles at the summit is greater than the third.

Find the locus of a point which is equidistant from three lines which meet at a point.

9. T is a point exterior to a conic section, and L, N are its orthogonal projections upon a fixed focal chord and the directrix respectively; if the ratio $SL : TN$ be equal to the eccentricity, show that T lies on one or other of the tangents at the extremities of the focal chord.

Deduce from this theorem a construction for drawing tangents to a conic from a given point T.

10. Take two points A and B an inch apart. Three conics have A for vertex, B for focus, and eccentricities 0·2, 1, 5 respectively. Determine a few points on each curve and draw the curves freehand.

11. Show that the locus of the middle points of any system of parallel chords of a conic is a straight line which meets the directrix on the straight line through the focus at right angles to the chords.

If a tangent be drawn parallel to any chord of a conic, prove that the portion of it terminated by the tangents at the ends of the chord is bisected at its point of contact.

12. Prove that one quarter of the rectangle between any focal chord and the latus rectum is equal to the rectangle between the segments of the chord determined by the focus.

Enunciate some other proposition concerning the length of the latus rectum which is true for the general conic.

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V. PART II. SECOND PAPER.

[*Full marks will be given for about* three-quarters *of this paper. The diagrams supplied are to be themselves used, and not pricked off into your book.*

A body in a vacuum falls to the earth with an acceleration of 32 *foot-second units.*]

1. Show how to resolve a force, represented by a given segment of a given straight line, into components, whose lines of action are parallel to two given straight lines.

If the direction of one of the components of the given force is given, but not its magnitude, and the magnitude of the other component is given, but not its direction, construct the magnitude of the former component and the line of action of the latter.

2. State (without proof) the conditions of equilibrium of a system of forces acting in one plane at a point.

Two small smooth rings of equal weight are free to slide on a thin circular wire, which is fixed in a vertical plane, and are connected by an inextensible thread, the length of the thread being less than the diameter of the circle. Prove that, if the system rests with the thread in a state of tension, the line of the thread must be horizontal.

3. AB represents a uniform bar of weight 11 lbs. and length 3·2 inches which rests at an angle of 50° on a rough horizontal plane at A and against a smooth peg at C, distant $2\frac{1}{4}$ inches from A. Find the magnitude of the reaction of the plane on the bar.

4. Prove that the sum of the moments of two like parallel forces about any point in the plane through their lines of action is equal to the moment of their resultant about the same point.

A uniform bar, of weight 5 lbs. and length 22 inches, is supported by two vertical cords attached to its ends, and two weights of $5\frac{1}{2}$ lbs. each are attached to it at points distant 8 inches and 18 inches from the same end. Find the tensions of the two cords.

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5. A circular disc, lying on a smooth table, has strings attached at A and B, which are pulled with forces represented by the lines AC and BD (an inch representing one poundweight). A third string is to be attached and pulled so as to keep the slab at rest. Determine a point E at which it may be attached, and represent by a line EF the magnitude and direction of the pull.

Also write down the magnitude of the pull.

$AC = 2{\cdot}44$ inches, $BD = 3{\cdot}18$ inches, and the angle between AC and BC $= 65°$.

6. Draw a figure of that system of pulleys in which the same cord passes round all the pulleys, and find the mechanical advantage of the system.

If a force of 50 poundsweight supports an attached mass of 220 lbs., and a force of 25 poundsweight supports an attached mass of 95 lbs., what force will be required to support an attached mass of 315 lbs., and what is the weight of the lower block?

7. A body moves from rest with uniform acceleration. From O is measured along OA a length OP representing any time elapsed. OB is drawn at an angle of $22\frac{1}{2}°$ with OA so that $\tan BOA$ represents the acceleration, and PQ is drawn perpendicular to OA to cut OB in Q. Show that PQ represents the speed at the time OP, and the area OPQ the space described in that time.

If the scales are chosen so that half an inch along OA represents a second, and an inch perpendicular to OA represents a speed of one foot per second, find the speed acquired in ten seconds.

8. Prove that, when a point moves with uniform acceleration, its average velocity over any interval of time is half the sum of its velocities at the beginning and end of the interval.

A train runs from rest at one station and stops at the next. During the first quarter of the journey the motion is uniformly accelerated, and during the last quarter it is uniformly retarded, the middle portion being performed at a uniform full speed. Prove that the average speed of the train is $\frac{3}{8}$ of the full speed.

9. Two bodies of equal mass, each m lbs., are attached to an inextensible thread; one of them is placed on a smooth table, and the other hangs over the edge; find the acceleration.

When the system has been moving for one second, the body on the table not having reached the edge, another body of equal mass, which was

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at rest on the table, becomes attached to it in such a way that the compound body moves in the line of the thread. Find the impulse of the tension in the thread.

10. A body which weighs 10 lbs. is observed to move horizontally with an acceleration of 4 foot-second units; find the magnitude and direction of the force applied to it.

If the body hangs from the roof of a railway carriage by equal strings attached to two points one directly in front of the other so that the strings are at right angles, and if the acceleration of the train is 4 foot-second units in the direction of motion, find the ratio of the tensions of the strings.

11. A particle is projected from a given point with velocity V, in a direction making an angle α with the horizontal; find the range on a horizontal plane through the point of projection.

A fielder 300 feet from the wicket throws in the ball with a speed of 100 feet per second and at an elevation of 30°. Will the ball return to the level of his hand before or after passing the wicket? The air-friction may be neglected.

12. A weight is attached to a cord of given length, of which the upper end is fixed, and the cord is held so as to make with the vertical an angle α. Find the velocity with which the weight must be projected horizontally in order that it may describe a horizontal circle uniformly.

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VOLUNTARY MATHEMATICS.

VI. PART III. First Paper.

[*Full marks will be given for about* two-thirds *of this paper.*]

1. C is the centre of an ellipse, and A, B extremities of the major and minor axes. If PN be the ordinate of P, and PG the normal, prove that

$$NG : CN :: CB^2 : CA^2.$$

2. Define an hyperbola in reference to a focus and a directrix, and thence establish the property enjoyed by the focal vectors of any point on the curve.

Given two points on a parabola and the direction of the axis, find the locus of the focus.

3. Investigate the equation to a straight line in the form

$$x \cos \alpha + y \sin \alpha = p.$$

Show that the three straight lines

$$x \cos \alpha + y \sin \alpha = p$$
$$x \cos \beta + y \sin \beta = q$$
$$x \cos \gamma + y \sin \gamma = r$$

will pass through the same point if

$$p \sin(\beta - \gamma) + q \sin(\gamma - \alpha) + r \sin(\alpha - \beta) = 0.$$

This condition being satisfied, find the co-ordinates of the point.

4. Prove that the equation

$$p\{(x-\alpha)^2 + (y-\beta)^2 - r^2\} + q\{(x-\gamma)^2 + (y-\delta)^2 - s^2\} = 0$$

denotes a circle.

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Find its radius, and show that its centre divides the line joining the centres of the circles

$$(x-\alpha)^2+(y-\beta)^2=r^2,\ (x-\gamma)^2+(y-\delta)^2=s^2$$

in the ratio $p:q$.

5. Under what circumstances does an equation of the second degree denote a parabola?

Find the direction of the axis of the parabola

$$\cos^2\alpha\, x^2+\sin 2\alpha\, xy+\sin^2\alpha\, y^2+2gx+2fy+c=0$$

and the co-ordinates of its vertex.

6. Two concentric circles are described about a fixed point C as centre, and any radius vector CPQ cuts them in P and Q. If a rectangle $PAQB$ be described about PQ as diagonal, so that PA, PB are in fixed directions, show that as the direction of CPQ varies A and B move upon ellipses.

Show the ellipses in a figure.

7. Tangents are drawn from the point (p, q) to the conic $ax^2+by^2=1$.

(i.) Find the equation of the tangents.

(ii.) Determine the conditions that must be satisfied by the numbers p, q, if the tangents are to include an angle α, and give a geometrical interpretation.

8. Give the complete specification of a couple, and show how couples may be compounded.

A circular disc of weight W and radius a is suspended horizontally by 3 equal vertical strings of length b attached symmetrically to the perimeter. Find the magnitude of the couple required to keep it twisted through a given angle θ.

9. A weight can be sustained upon a rough inclined plane by a force P acting along the plane or by a force Q acting horizontally; show that the weight is $\dfrac{PQ}{(Q^2\sec^2\phi-P^2)^{\frac{1}{2}}}$ where ϕ is the angle of friction.

10. Define relative velocity and relative acceleration. A stone is projected from the top of a tower with velocity v and at an angle of elevation α, and, at the same instant, another stone is dropped from the same place. Find the relative velocity and acceleration of the first stone with respect to the second.

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11. Define the poundal, the dyne, and the horse-power. Find the speed of a train, in miles per hour, given that the locomotive of 50 horse-power works against a resistance of 10,000 poundals.

12. Calculate the length of a seconds pendulum at a place where a body falls to the earth with an acceleration of 32·19 foot-second units.

Given that the force of gravity varies inversely as the square of the distance from the earth's centre, find an approximate formula for the number of seconds a day lost by a seconds pendulum when taken to a height h, which is small compared with the earth's radius.

If h be 5 miles, and the earth's radius 3956 miles, show that the loss is about 109 seconds a day.

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VII. PART III. SECOND PAPER.

[*Full marks will be given for about* two-thirds *of this paper. The acceleration due to gravity may be taken to be* 32 *foot-second units.*]

1. From a point T two tangents TP and TQ are drawn to touch a parabola, whose focus is S, at P and Q, and T' is any point on TS produced beyond S. Prove that the angles PTQ, PST', QST' are all equal.

2. From the foci S and S' of an ellipse perpendiculars SY and $S'Y'$ are drawn to the tangent at any point P. Prove that Y and Y' lie on the auxiliary circle.

If the perpendicular from the centre on the tangent at P and a focal radius SP (produced if necessary) meet in R, then SR is equal to half the major axis.

3. Find an expression for the length of the perpendicular from the point (h, k) to the straight line $ax+by+c=0$.

Find the locus of a point which moves so that the perpendiculars drawn from it to the two straight lines

$$3x+4y=5, \quad 12x-5y=13$$

are equal.

4. Explain how a single equation can represent two straight lines.

Form a single equation to represent the two straight lines

$$3y-2x=1, \quad 5y+4x=2.$$

Also find the equation of the two straight lines joining the origin to the points where these lines are met by the line $y=\frac{2}{11}$.

5. Two circles are drawn to touch both the lines $y=0$, $y=x\tan 2\alpha$, and to pass through the point (h, k); prove that the distances of the points of contact from the origin are the roots of the equation

$$\xi^2-2(h+k\tan\alpha)\xi+h^2+k^2=0.$$

Find the equation of the common chord of the two circles.

6. Find the equation of the tangent to the parabola $y^2=4ax$ at the point (x', y').

From a fixed point (h, k) a line is drawn at right angles to the tangent to the parabola at P, and meeting the diameter through P in Q. Prove that the locus of Q for different positions of P is the rectangular hyperbola

$$y(x+2a-h)=2ak.$$

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7. Form the equation of the chord of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ joining the points of which the eccentric angles are θ and ϕ.

Prove that, if P is the point for which the eccentric angle is $\frac{1}{2}(\theta+\phi)$, and the line joining P to either focus meets the chord in Q, then

$$QP=a\left(1-\cos\frac{\theta-\phi}{2}\right).$$

8. Prove that, if three forces in one plane acting on a rigid body are in equilibrium, their lines of action meet in a point or are parallel.

A uniform square lamina is supported, so that its plane is vertical and one of its corners is in contact with a fixed smooth plane of inclination α, by a force applied at the opposite corner in a direction parallel to the line of slope of the inclined plane. Prove that the diagonal which passes through the point of contact makes with the vertical an angle 2α.

9. Two particles of masses, m, m', are connected by an inextensible thread; m is placed on a rough inclined plane, and a thread tied to it lies along a line of slope, passes over a pulley at the top of the plane, and supports m' which hangs free. Find the condition that m should move (1) upwards, (2) downwards, and the acceleration in each case. [Take μ for the coefficient of friction.]

10. A body which weighs one ton is supported by a wire rope which weighs two pounds per foot. Find the tension at the upper end of the rope when the body is just above the bottom of a mine 200 feet deep.

Find also the work done in lifting it gradually to the surface.

11. A smooth sphere lying at rest on a horizontal table is struck by an equal sphere moving on the table so that just before the impact the direction of its motion makes an angle α with the line of centres. Taking e for the coefficient of restitution, find the directions of motion of both spheres after impact.

12. Prove that the velocity of a projectile at any point is that due to falling from the directrix of its path.

Prove also that, if V is the velocity of projection and α the initial angle of elevation, the equation of the path referred to horizontal and vertical axes through the point of projection is

$$gx^2=2V^2\cos^2\alpha\,(x\tan\alpha-y).$$

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ANSWERS.

JUNE, 1891.

II. Algebra.

1. $(x-1)^4-(x-1)^2+1$.

2. $\frac{x^2+1}{x^2+2}$; $(x-14)(x-12)$, $(x^2+xy+y^2)(x^2-xy+y^2)$.

4. $2x^2+3x+4$.

5. (i.) $\frac{5}{7}$. (ii.) $1\frac{1}{2}$, $\frac{5}{6}$.

6. 654321, 1562202.

7. (i.) $\frac{b^2}{a}$, $-\frac{1}{a}$. (ii.) $x=\frac{3}{2}$, $-\frac{1}{2}$; $y=-\frac{1}{2}$, $\frac{3}{2}$.

8. $6\frac{1}{2}$ and 5 sq. ft.

10. $1+\frac{1}{3}+\frac{1}{9}+\ldots$; 10, 12, ... 60, infinity.

11. m, 5th and 6th terms $=\frac{481250}{256263}$.

12. 4·60898, £100,000.

III. Trigonometry.

1. $\frac{13}{12}$, $\pm\frac{\sqrt{13}}{3}$, $\frac{169}{120}$.

3. $\tan(n-1)A$.

8. 56° 30′ 56″ and 123° 29′ 4″.

9. $\sqrt{2(x+1)}$.

10. £19962. 13*s*. 6*d*.

11. 1221·88 cubic inches.

IV. Statics and Dynamics.

2. $\frac{1}{2}W$, $\frac{\sqrt{3}}{2}W$.

3. 7 oz., $9\frac{1}{7}$ inches from A.

4. $\frac{39+4\sqrt{6}}{75}a=\cdot 6506a$.

5. Round C in direction BOA.

7. $34\frac{2}{5}$.

8. $\frac{2}{3}$ f.s.s.

10. $\frac{1}{4}$ sec.

11. $9\frac{3}{13}$ f.s.

12. 350 ft.

V. Pure Mathematics (1).

2. SP = abscissa, or latus rectum.

8. $x(a\sin\alpha+b\sin\beta)-y(a\cos\alpha+b\cos\beta)=0$,
$x(a\sin\alpha-b\sin\beta)-y(a\cos\alpha-b\cos\beta)=ab\sin(\alpha-\beta)$.

12. $\theta=\frac{\pi}{2}$, $r\cos\theta=\frac{2eL}{1-e^2}$.

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VI. Pure Mathematics (2).

2. $\frac{4308}{128}$.

3. $\frac{1}{15}$.

4. $a+\dfrac{1}{2a+1+}\ \dfrac{1}{2a+}\ \dfrac{1}{2a+1+}$.

5. 1680.

6. (i.) $x=a,\ \frac{a}{2}\{\sqrt{2\sqrt{3}}+\sqrt{3}-1\},\ -\frac{a}{2}\{\sqrt{2\sqrt{3}}-\sqrt{3}+1\},\ \frac{a}{2}\{\sqrt{-2\sqrt{3}}-\sqrt{3}-1\},$

$-\frac{a}{2}\{\sqrt{-2\sqrt{3}}+\sqrt{3}+1\};\ y=-a,\ \frac{a}{2}\{\sqrt{2\sqrt{3}}-\sqrt{3}+1\},\ -\frac{a}{2}\{\sqrt{2\sqrt{3}}+\sqrt{3}-1\},$

$\frac{a}{2}\{\sqrt{-2\sqrt{3}}+\sqrt{3}+1\},\ -\frac{a}{2}\{\sqrt{-2\sqrt{3}}-\sqrt{3}-1\};$

(ii.) $x=\dfrac{c\cos\overline{\alpha-\beta}}{\cos\beta},\ y=\dfrac{c\cos\overline{\alpha+\beta}}{\cos\beta}$.

11. (i.) $1-\log_e 2$; (ii.) $\frac{1}{2}-\frac{\pi}{8}$; (iii.) $1-\frac{1}{2}(e^{\sqrt{-1}}+e^{-\sqrt{-1}})$.

VII. Mechanics.

1. 37·81 lbs.

2. ·458$\frac{3}{3}$, 98·66 lbs.

3. $\sin\theta<\frac{Q}{P}$.

5. It slips.

7. 30°.

10. At a point y from the end, the tension $=\frac{y}{2l}\left(1-\frac{x}{l}\right)W$.

11. 667 yards.

12. $\sqrt{rg}\,.\sin 2\theta$, where $\theta=\angle$ its distance from O makes with vertical.

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NOVEMBER, 1891.

II. ALGEBRA.

1. 0. 2. $2x^7-7x^2+5$, $a^4-3a^3b+6a^2b^2-3ab^3+b^4$.
3. $x-1$, $x^{15}+2x^{14}+3x^{13}+4x^{12}+\ldots$. 4. x^2+2x+1.
5. -3. 6. (i.) $\frac{3}{4}$; (ii.) $x=5\frac{52}{105}$, $y=2\frac{67}{105}$.
8. (i.) $3(3\pm\sqrt{17})$; (ii.) $\pm 6\sqrt{\dfrac{a+1}{4a^2-4a-26}}$. 9. 1254.
10. $n(n-1)(n-2)(n-3)$. 12. 519·78.

III. TRIGONOMETRY.

3. $\frac{84}{13}$. 5. Two.
9. 131° 4′ 18″·6. 10. 52·2405 ft.
11. 96 cub. in., 138·528 sq. in. 12. $2\frac{113}{336}$ tons.

IV. STATICS AND DYNAMICS.

1. 0. 2. 5 : 2. 3. $\sqrt{3}-1$.
5. 45°. 6. 1 lb. 7. $\frac{3}{4}$ and $1\frac{1}{2}$ secs.
8. $2\frac{2}{3}$ miles. 9. (i.) $\sqrt{3}:1$; (ii.) equal. 10. $32\sqrt{3}$.
11. $21\frac{9}{11}$, 360 feet. 12. $m=M$.

V. PURE MATHEMATICS (I).

2. Sides $7x+y=11$, $3x-y+1=0$, $x+3y+7=0$;
bisectors $99x+77y+71=0$, $7x-9y=37$. 3. A straight line.
4. (i.) A circle; (ii.) an ellipse, parabola, or hyperbola according as the constant is $>$, $=$, or $<$ the distance between the centres.
5. Two. 6. When the distance of the point from the origin $> \dfrac{OS^2}{BC}$
8. 8, $2\sqrt{3}$. 9. A parabola.

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VI. Pure Mathematics (2).

1. (i.) 6, -7; (ii.) $x = \pm a$, $\pm\frac{2a-b}{\sqrt{3}}$; $y = \pm b$, $\pm\frac{a-2b}{\sqrt{3}}$.

2. $\frac{1}{6}n(n+1)(n+2)$. 5. 89.

6. $\sin(A+2B)$ is positive, $\cos(A+2B)$ is negative.

8. $\frac{a\sin\gamma}{\sin(\alpha+\beta)}$. 9. 9·65146, 20·5309.

VII. Mechanics.

1. 5·632, 4·792. 2. The c.g. is 3″·19 from A; 12 oz., 17 oz.

3. Tension, $\frac{5}{6}W$. Pressures on the pegs, $\frac{\sqrt{5}}{3}W$, $\frac{2\sqrt{5}}{3}W$, $\frac{4}{3}W$.

7. In $1\frac{1}{4}$ secs., 37 ft. from lower point.

8. $\frac{P-Q}{P+Q}\cdot g\cos\theta$, $\frac{2PQ}{P+Q}\cos\theta$. 10. (i.) $\sqrt{5ag}$; (ii.) $\sqrt{(5a-3b)g}$.

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ANSWERS.

JUNE, 1892.

II. Algebra.

1. $a^6 - a^5b + 9ab^5$. 2. $x^3 + p^2x^2 + pqx + q^2$.

3. $2x^2 + 3x - 5$; $(2x+5)(2x-5)(x+1)(x-1)$, $(2x+5)(x-1)(x^2-x-3)$.

4. (i.) $3 \div (2 - 5x + 6x^2 - 4x^3 + x^4)$; (ii.) $a+b$; (iii.) $\frac{1}{a}$.

5. $4(a+b)$. 6. (i.) $10\frac{1}{3}$, -3; (ii.) $\frac{2}{3}$, 2.

7. $c^4x^2 - 4bc^2x + 4b^2 = 0$. 8. 40.

9. $9\frac{2}{3}$, $10\frac{1}{3}$, sum 460; 1 and 9. 10. 35, 56.

11. 12th term, $-4368a^{-6}$; middle term, 12870. 12. 18.

III. Trigonometry.

1. At 10 min. 40 sec. after 1 o'clock. 4. 75°, 15°.

9. 82329·3 sq. ft., 100·647 ft. 11. 1886·32 sq. ft.

12. $13\frac{17}{85}$ sq. ft.

IV. Statics and Dynamics.

1. 173·2, 200, 173·2. Resultant $= 2AK$, where K is mid-point of CD.

3. 12 lbs. 4. $\frac{2}{3}$ inch.

5. α must be less than 120°. 6. $\frac{3P}{\sqrt{10}}$.

8. 72 : 245. 10. 420.

11. Greatest height, 25 ft.; range, $100\sqrt{3}$ ft.; velocity, $16\sqrt{21}$ f.s.; distance, $32\sqrt{19}$ ft.

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V. Pure Mathematics.

2. $1-3x+3x^2-3x^4+3x^5-3x^7+3x^8-\ldots$. **3.** 8 and 4. **10.** a^2.

11. (i.) $x+2y=a$, $2x-y=a$; (ii.) $4x+3y=0$, $3x-4y=0$; a right angle.

14. $7y^2+24xy-24ax-6ay+15a^2=0$. Centre, $\left(-\frac{a}{3}, a\right)$; directrix, $12x-9y+29a=0$.

VI. Mechanics.

1. $OB : OA$. **2.** Length, 1·364 inches; tension, 63·11 oz.

3. At B, 270·4 lbs.; at E, 375 lbs.

4. It tilts first. **6.** A parabola.

9. Velocities, o, u; distance apart, $a-ut$. Taking point of projection as origin, and the axes horizontal and vertical, equation to C.G. is $(2x-a)^2=\frac{u^2}{16}y$. **11.** $94587\frac{3}{56}$ ft.-lbs.

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NOVEMBER, 1892.

II. Algebra.

1. $8x^3 - 8bx^2 + 12abx - 27a^3$, $8a^3 + 4a^2(bc)^{\frac{1}{2}} - 2abc + (bc)^{\frac{3}{2}}$.

2. $3a^3 - 2a^2b - \frac{2}{3}ab^2 + \frac{1}{9}b^3$.

3. The ascending order is $\frac{(n-1)a}{nb}$, $\frac{na}{(n+1)b}$, $\frac{(n+1)a}{nb}$, $\frac{na}{(n-1)b}$.

4. (i.) a^{m-2}; (ii.) $\frac{2x(x+2a)}{a(x-2a)}$; (iii.) $a^2 - ab - b^2$.

5. 1, $1+\sqrt{2}$.

6. (i.) -5; (ii.) $x = a, -\frac{a}{2}$; $y = \frac{a^2}{b}, -\frac{2a^2}{b}$; (iii.) 14.

7. $x^2 + 2x - 24 = 0$, -14.

8. $\frac{4}{3}a - \frac{2}{3}b$, $\frac{5}{3}a - b$, $2a - \frac{1}{3}b$, $\frac{7}{3}a$, $\frac{8}{3}a + \frac{1}{3}b$; 10 terms.

9. 3360, 126. 10. $1 - 2x - 2x^2 - 4x^3 - 10x^4$. 11. 4·91956.

III. Trigonometry.

1. Cosine $\frac{25}{37}$, tangent $\frac{12}{35}$, cotangent $\frac{35}{12}$, secant $\frac{37}{35}$, cosecant $\frac{37}{12}$.

4. $2\sin\frac{A}{2} = \sqrt{1-\sin A} - \sqrt{1+\sin A}$. 6. ·951.

8. 29·7443. 10. 322·847 ft.

11. 190·765 sq. ft. 12. 7·432 ft.

IV. Statics and Dynamics.

2. $1\frac{17}{19}$ lbs. 4. 5 lbs.

5. Perpendicular to BC; 9 in. from B. 6. Nearly $6\frac{39}{50}$ ft.

8. $7u$. 9. 62 ft.

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V. Pure Mathematics.

1. 1210.

2. $\dfrac{1}{a+\dfrac{1}{2a+}\,\dfrac{1}{2a+}\cdots}$.

3. $\frac{1}{2}$, $\frac{4}{7}$.

4. 3 : 4 : 5.

10. An ellipse, centre (a, o), semiaxes a, $2a$.

11. $y = a$, $4x - 3y + 3a = 0$.

14. $4x + y = 0$.

VI. Mechanics.

1. 5·023 lbs.

2. 1 cwt.

3. ·6248 cwt.

6. 30° to horizontal.

7. $t\{v^2 - vg\sin\alpha \,.\, t + \frac{1}{2}g^2t^2\}^{\frac{1}{2}}$.

8. $g\sin\alpha\cos\alpha$, $g\sin^2\alpha$.

11. $\dfrac{3388\,Mv^2}{45(P-Mp)}$, $\dfrac{3388\,Mv^2}{45(R+Mp)}$.

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ANSWERS.

JUNE, 1893.

II. Algebra.

1. $(2x-5y)(3x-4y)$, $(x^2-3xy+y^2)(x^2+3xy+y^2)$,
$(x-1)(x+1)(x^2+x+1)(x^2-x+1)$.

2. $5xy(x+y)$.

3. x^2-2x+5.

4. (i.) 0; (ii.) 4; (iii.) $1\frac{49}{60}$.

5. (i.) $3x^2-7x-2$; (ii.) $\sqrt{\frac{11}{2}}-\sqrt{\frac{7}{2}}$.

7. (i.) 7; (ii.) 7; (iii.) $x=\pm 3, \pm 2$; $y=\pm 2, \pm 3$.

8. $(a-b+c)x^2-2(a-c)x+a+b+c=0$.

9. $-2, 0, 2, 4, 6$.

11. 8th term $=-\frac{5049}{5^8}x^7$.

12. ·6309.

III. Trigonometry.

1. 57·2727, 36·0818.

2. $\frac{240}{289}$, $\frac{161}{289}$, $\frac{240}{161}$.

7. 180·24 ft.

8. 37° 52′ 27″.

11. 1 ft.

12. 27·7128 square inches, 7·542 cubic inches.

IV. Statics and Dynamics.

1. Resultant is $\sqrt{57}P=7{\cdot}55P$, making an $\angle \tan^{-1}\frac{4}{\sqrt{3}}=66\frac{1}{2}°$ with OB.

2. $\frac{8}{5}W$, $\frac{8}{5}W$.

3. At A $96\frac{3}{7}$ lbs., at B $53\frac{4}{7}$ lbs.

4. At mid-point of BC; $8\frac{1}{3}$ lbs., $6\frac{2}{3}$ lbs.

6. 16 lbs., 40 lbs.

7. $x=2016$, $y=224$.

8. $4\frac{6}{11}$ f.s.s.

9. A is $\frac{1}{2}$ ft. away when B gets to C.

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V. Pure Mathematics.

2. $(1+y+xy)(1+xy+x^2y)$.

3. $\frac{1}{2}\{\gamma+\delta\pm\sqrt{(\gamma+\delta)^2-4\alpha\beta}\}$.

4. $\frac{1}{6}n(n-1)(n+4)$.

5. $2^{2n-1}-\dfrac{\lfloor 2n}{2(\lfloor n)^2}$.

9. $\dfrac{\sin\alpha}{1-2x\cos\alpha+x^2}$.

10. O is the focus. If OT be the perpendicular from O on the tangent and OT be produced to T' so that $OT'=2OT$, a line through T' making angle α with the tangent is the directrix.

13. $\dfrac{a^2-b^2}{a^2}\cdot x'$, $\dfrac{b^2-a^2}{b^2}\cdot y'$.

14. $Ax+By=0$; $Bx+Cy=0$. If these lines coincide $B^2=AC$, and the curve becomes two coincident parallel lines.

VI. Mechanics.

1. 3·6 lbs., $\frac{8}{3}BC$ from B.

2. 57·7 lbs.

3. $\dfrac{AB}{2}(\cot\phi\tan\alpha-1)$.

4. 21.

6. S.W. wind, whose speed equals that of vessel.

10. $37\frac{1}{2}$ miles an hour; $6\frac{5}{12}$ min., $2\frac{1}{192}$ miles.

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NOVEMBER, 1893.

II. Algebra.

1. x^2+3y^2.

2. $x(x-4)(4x-7)$, $(y+3)(y-3)(y^2+20)$,
$(a^2+3b^2)(a^2-3ab+3b^2)(a^2+3ab+3b^2)$. 3. $5x^2-6x-7$.

4. (i.) 0; (ii.) $\frac{a}{b}-1-\frac{b}{a}$; (iii.) $\frac{3}{5}$. 5. (i.) $2x^2+3x-5$; (ii.) $6-\sqrt{5}$.

6. (i.) $\frac{a^2+ab+b^2}{a+b}$, $\frac{ab}{a+b}$; (ii.) 0, 7, $-2\frac{1}{21}$; (iii.) $\frac{2}{3}$, $\frac{1}{3}$.

11. The 6th and 7th terms each $= \frac{6188}{243}$.

III. Trigonometry.

1. 1·309 ft. 2. -7, $-\frac{17}{31}$. 3. (i.) $2\cos^3 A$; (ii.) $\sqrt{2}\tan\theta$.

8. 2·52982. 9. $\frac{1}{2}b^2\sin 2A$.

11. $151\frac{83}{112}$ cubic feet. 12. $382674\frac{2}{7}$ square miles.

IV. Statics and Dynamics.

2. 45° 49′ 21″. 4. $-\frac{P}{2}$, $\frac{P}{\sqrt{2}}$, $\frac{P}{2}$. 6. $19\frac{5}{9}$.

7. After $\frac{5}{8}$ secs. and $2\frac{1}{2}$ secs. At height of $29\frac{19}{64}$ ft.

8. $\frac{1}{60}$ ounce-weight. 9. $\frac{Qg}{P+Q}$, $\frac{PQ}{2(P+Q)}$.

V. Pure Mathematics.

2. $\frac{30}{n}\{\sqrt{1+4n^2}+1\}$, $\frac{30}{n}\{\sqrt{1+4n^2}-1\}$.

3. $(1+x^n)^{-1}\{(1+x^n)^{\frac{1}{n}}-x\}^{-1}$, $\{(1+x^n)^{\frac{1}{n}}-x\}^{-1}$.

5. $2\log_a e\left\{\frac{x-1}{x+1}+\frac{1}{3}\left(\frac{x-1}{x+1}\right)^3+\frac{1}{5}\left(\frac{x-1}{x+1}\right)^5+\dots\right\}$.

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6. $90°$, $30°$.

7. $\frac{1}{4}\sqrt{\left\{\frac{s_1}{3}(6s_1s_2 - 8s_3 - s_1^3)\right\}}$, 6.

8. $3\frac{3}{4}$ ft.

9. $2x - \sqrt{6}y + 1 = 0$.

11. $x^2 + y^2 - 3x + 2 = 0$, $2x^2 + 2y^2 - 5x - \sqrt{3}y + 3 = 0$,
$2x^2 + 2y^2 - 7x + \sqrt{3}y + 6 = 0$.

12. $\frac{a^2}{x'}$, $\frac{b^2}{y'}$.

VI. Mechanics.

2. $45\frac{1}{2}$ lbs.

3. $5\frac{1}{20}$ tons.

5. $\tan^{-1}2\mu$.

7. The focus is a point $30\frac{1}{4}$ ft. below the point from which the ball was dropped; the directrix is a line parallel to the direction of the train $30\frac{1}{4}$ ft. above the roof.

8. (i.) 1200 lbs.; (ii.) 34800 lbs.

9. (i.) $1:3$; (ii.) $1:2e+1$.

10. $5\frac{5}{19}$ inches.

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ANSWERS.

JUNE, 1894.

II. Arithmetic.

1. $22\frac{5}{7}$.
2. ·0016.
3. £74. 11*s*. $10\frac{1}{2}$*d*.
4. £1. 13*s*. $10\frac{1}{2}$*d*.
5. $\frac{7}{40}$.
6. £30. 12*s*. 3*d*.
7. 660.
8. $2\frac{1}{2}$ inches.
9. 12.
11. *A* had originally £1380, *B* £3220.
12. 6 days.
13. 15.
14. £5.

III. Algebra.

1. $x^3-3x^2+11x-8$.
2. $(x-2y)(x^2+2xy+4y^2)$, $(x-8y)(x-16y)$, $(x^2+xy+y^2)(x^2-xy+y^2)$.
3. x^2-7x+2.
4. $5(a^2+b^2+c^2-bc-ac-ab)$.
5. (i.) n^2+3n+1; (ii.) $\sqrt{13}+\sqrt{11}$.
6. (i.) $\frac{a+c}{2}$; (ii.) 4, -7.
7. $x^2+3px+2p^2+q=0$.
8. 16 and 40 years.
9. 12, $\frac{1}{12}$.
12. 2·5854607, ·1602526, 5·82.

IV. Trigonometry.

1. $343^\circ\ 38'\ 10\frac{10}{11}''$.
2. $-\frac{2}{3\sqrt{7}}$.
3. 49·177 ft.
4. $-\frac{1}{\sqrt{2}}$, $-\frac{\sqrt{3}}{2}$, -1, $\frac{2}{\sqrt{3}}$.
8. $21^\circ\ 47'\ 12\frac{1}{2}''$, $38^\circ\ 12'\ 47\frac{1}{2}''$, 120°.
9. $\left\{\frac{x^2\sin^2\alpha}{\sin^2(\beta-\alpha)}-\frac{2xy\sin\alpha\cos\beta\sin\gamma}{\sin(\beta-\alpha)\sin(\gamma-\beta)}+\frac{y^2\sin^2\gamma}{\sin^2(\gamma-\beta)}\right\}^{\frac{1}{2}}$.
10. 60°, $44^\circ\ 49'\ 20''$.
11. 114582 sq. ft.
12. 4·0825 inches.

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V. Statics and Dynamics.

1. (i.) Equilibrium impossible; (ii.) Parallel to sides of a right-angled triangle.

2. $\sqrt{\{1440+304\sqrt{2}+180\sqrt{3}-144\sqrt{6}\}}$, $\tan^{-1}\dfrac{29+8\sqrt{2}}{10+9\sqrt{3}-8\sqrt{2}}$ with OA.

3. (i.) $216\frac{2}{3}$ lbs. (ii.) 200 lbs.

5. 5 lbs.-weight at distance 16″ from greater force.

6. $\frac{25}{36}$ inch from centre. 8. Ratio is 11 : 10.

9. At height 3520 ft., 11 secs. after the first was fired.

10. 3 lbs.-weight, or 96 poundals.

11. Range 7500, velocity $20\sqrt{241}$ in direction $\tan^{-1}\frac{4}{15}$ to horizontal.

VI. Pure Mathematics.

2. ·000762588, ·000000566236. 3. $\frac{25}{27}$ secs., $1\frac{2}{13}$ secs.

9. $\dfrac{x}{p}+\dfrac{y}{q}=2.$

10. If the equation to the circle be $x^2+y^2+Ax+By+C=0$, $(A+B)^2>8C$.

11. $x^2+y^2-5ax=0$; $\left(a, 2a\right)$, $\left(\dfrac{9a}{2}, -\dfrac{3a}{2}\right)$, and $\left(a, -2a\right)$, $\left(\dfrac{9a}{2}, \dfrac{3a}{2}\right)$.

13. $x^2+y^2+2xy\dfrac{a^2-b^2}{a^2+b^2}=a^2-b^2.$

VII. Mechanics.

2. $n\pi$. 3. $\dfrac{3b}{\sqrt{2}}$. 5. $\frac{3}{4}l$. from end of the bar.

8. 20,000 ft., 25·5 ft. 9. $\frac{2}{3}W$.

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NOVEMBER, 1894.

II. Arithmetic.

1. $\frac{1}{8}$.
2. 440·82.
3. $4\frac{1}{2}$ per cent.
4. 4 cwt. 1 qr. 14 lbs.
5. $\frac{1}{60}$.
6. £200. 10*s*. $2\frac{3}{4}$*d*.
7. 47.
9. £80.
10. 3 p.m.
12. Latter.
13. £194. 5*s*., £224. 14*s*.
14. $37\frac{1}{2}$ miles.

III. Algebra.

1. $x^4+ax^3-a^3x-a^4$.
2. $-\dfrac{(x+y-z)^2}{2yz}$.
4. $-(a+b+c)$.
5. $x=-3,\ y=1\frac{1}{2},\ z=4$.
6. $-\frac{1}{2},\ \pm 3$.
7. $-1,\ -\frac{1}{2}$.
8. Series is $3+2\frac{1}{2}+2+1\frac{1}{2}+\ldots$.
9. $1+\frac{1}{3}x+\frac{2}{9}x^2+\frac{14}{81}x^3+\ldots+\dfrac{1\cdot 4\cdot 7\ldots(3n-5)}{3^{n-1}\lfloor n-1}x^{n-1}+\ldots$.
10. 45 and 36 miles an hour.
11. 3·2101493, 2·30258.

IV. Trigonometry.

1. $57\frac{3}{11}°$.
3. 15°.
5. 20°.
8. $B=54°\ 56'\ 48''$, $C=83°\ 3'\ 12''$, $c=209{\cdot}174$; or $B=125°\ 3'\ 12''$, $C=12°\ 56'\ 48''$, $c=47{\cdot}2108$.
9. 109° 6′.
10. $3\frac{1}{7}$ sq. feet.
11. 12·143 sq. inches.

V. Statics and Dynamics.

1. $2AC$.
3. 100 lbs., 40 lbs.; 112 lbs., 28 lbs.
5. $\dfrac{W}{2},\ \dfrac{W}{\sqrt{3}}$.
6. 1.
7. 1024 ft., 16 seconds; 4 and 12 seconds.
9. 45°, 160 f.s.
10. 2 lbs.-weight, or 64 poundals.

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VI. Pure Mathematics.

4. 103923, percentage ·7725. **5.** 4.

6. 82° 25′ 22″ and 57° 34′ 38″.

7. $\frac{4}{p}\sin\frac{\phi''-\phi'}{2}\sin\frac{\phi''-\phi}{2}\sin\frac{\phi'-\phi}{2}$, ·000227633. **10.** $y-k=\sqrt{3}(x-h)$.

11. $2\sin^{-1}\frac{\rho}{\sqrt{\alpha^2+\beta^2}}$. **14.** $(\sqrt{2}-1)x^2-(\sqrt{2}+1)y^2=\frac{25}{2\sqrt{2}}$.

VII. Mechanics.

1. 7·89. **2.** $\frac{W\sqrt{3}}{2}$ making angle 60° with horizon.

3. $\sqrt{13}:2$. **5.** 14 lbs. **8.** 100 ft.

9. $3\frac{1}{2}$ seconds. **10.** $50\sqrt{2}$ f.s., $\tan^{-1}\frac{4}{3}$.

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ANSWERS.

JUNE, 1895.

II. Arithmetic.

1. $30\frac{13}{15}$.	**2.** ·22.	**3.** $7\frac{1}{2}$ years.
4. ·7875.	**5.** £74. 1*s.* $5\frac{1}{2}$*d.*	**6.** £555. 10*s.* 6*d.*
7. 325.	**8.** $6\frac{1}{4}$ tons.	**9.** 30.
11. *A*, £540; *B*, £675; *C*, £1200.		**12.** 3 per cent.
13. 55, 25.	**14.** £23. 14*s.*	

III. Algebra.

1. 0. **2.** (i.) $(x-1)^2(x+1)(x-2)^2(x^2-2)(x^2+2)^3$; (ii.) $x^8(x^8-y^8)$.

4. (i.) 8, 9; (ii.) x, $7\frac{2}{11}$; y, 3; (iii.) $x=\pm\frac{1}{2}$, $\pm\frac{4}{3\sqrt{3}}$; $y=\pm\frac{1}{3}$, $\mp\frac{7}{6\sqrt{3}}$.

5. *A*, 67; *B*, 28. **9.** $b^y=c^x$, 1·77245.

IV. Trigonometry.

1. ·8108936, 1·1934918. **2.** (i.) $\frac{15}{17}$; (ii.) $\frac{5}{\sqrt{34}}$.

3. $\frac{y}{x}$ and $\frac{x}{y}$. **9.** 53° 7′ $48\frac{1}{2}$″. **10.** 15 ft.

V. Statics and Dynamics.

1. 0. **4.** $\frac{1}{8}$ weight of lever. **5.** 10,000 lbs.

6. $\frac{1}{120}$ f.s., $\frac{13}{240}$ f.s. **7.** (i.) $\frac{3}{2}$ f.s.s.; (ii.) 2 f.s.s.

8. (i.) 2 f.s.s.; (ii.) $16\sqrt{3}$ f.s.; (iii.) $8\sqrt{3}$ sec. **9.** $\frac{11}{3000}$. **10.** 30°.

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VI. Pure Mathematics.

1. (i.) a^2, a; (ii.) $x = \pm\frac{1}{2}(\sqrt{3b^2-2a^2} \pm \sqrt{2a^2-b^2})$; $y = \frac{1}{2}(\sqrt{3b^2-2a^2} \mp \sqrt{2a^2-b^2})$.

5. 173·089 ft. **8.** 9, 6; −6, −4.

9. $11x + 3y = 30$, $3x - 11y = 20$.

10. Radius 1, points of contact, 1, 1 and 2, 2.

12. The co-ordinates of the point are $c \tan(\theta + \phi)$, $-c \tan(\theta + \phi)$.

14. $GK = \frac{1}{2}$ latus rectum.

VII. Mechanics.

1. 7 lbs. **2.** $33\frac{5}{24}$, $33\frac{19}{24}$ lbs.

3. Distance from centre = $\frac{1}{18}$ side of square. **4.** $\frac{28}{171}$.

5. 6 ft. **6.** 12 : 5. **7.** $\frac{7}{48}$ mile, 35·35 miles per hour.

9. 60 f.s., $\tan^{-1}\frac{3}{4}$. **10.** $W\left(\cos\alpha + \frac{v^2}{gL}\right)$, $W\left(\sin\alpha - \frac{v^2}{gL}\cot\alpha\right)$.

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NOVEMBER, 1895.

II. Arithmetic.

1. $\frac{8}{5}$. **2.** 1562·5. **3.** £531. 13*s*. 4*d*.

4. 31 lbs. 4 oz. 16 dwts. 16 grs. **5.** ·941$\dot{6}$. **6.** £1092. 15*s*. 8*d*.

7. 1155. **8.** £7. 15*s*. **9.** £25. 13*s*. 4*d*.

11. $1\frac{1}{2}$ inches. **12.** £17. 10*s*., £70, £157. 10*s*.

13. $6\frac{3}{7}$ secs., 204 yds. **14.** 7 per cent.

III. Algebra.

1. x^3-2x^2+4x+7.

2. (i.) $(x+7)(x+9)$; (ii.) $(y-a)(y+7a)(y-6a)$;
(iii.) $x(x+1)(x-1)(x+2)(x-2)(x+3)(x-3)$.

3. x^2-5x+1. **4.** $(a+b-c)(a-b+c)(a-b-c)$.

5. (i.) 6, 5; (ii.) $\frac{4}{5}$, $-\frac{3}{5}$; (iii.) $x=\pm 2, \pm 1$; $y=\pm 1, \mp 2$. **6.** £4000.

7. 51 miles an hour, $93\frac{1}{2}$ miles. **8.** $2\frac{179}{243}$, 45 terms.

9. $4(p^2-q)$. **10.** 136080.

11. 1, 1, $\frac{3}{2}$, $\frac{5}{2}$, $\frac{35}{8}$, $\frac{63}{8}$; greatest term $= 9289\frac{91}{125}$. **12.** ·418518.

IV. Trigonometry.

1. (i.) 144°; (ii.) $\frac{4\pi}{5}$. **2.** $\frac{3}{4}\sin^2 2A$. **3.** 10°.

4. $n\pi+\frac{\pi}{3}$, $n\pi+\frac{3\pi}{4}$. **9.** 205·4.

11. Area ·0403125 sq. inches; length, 6·1416 inches.

V. Statics and Dynamics.

1. 135°. **2.** Length, 20 inches; weight, 12 lbs.

3. Force, 2 lbs.; hinge action, $2\sqrt{5}$ lbs., making angle $\tan^{-1}\frac{1}{2}$ with AB.

4. $\frac{7\sqrt{3}}{10}a$ from vertex of lightest triangle.

5. Second system, 4 sheaves in upper, 3 in lower block, with the string fastened to lower block. 35 ft.

6. 10·6 miles an hour. **8.** 312 ft.; 64 f.s., 144 f.s.

9. $\frac{5\sqrt{2}}{6}$ seconds. **10.** $\frac{1}{8}$ range.

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VI. Pure Mathematics.

1. £5436. 2*s*. $5\frac{1}{2}$*d*.

2. *A* arrives 1 min. 24 secs. before *B*; $12\frac{1}{2}$ miles an hour.

3. ·47829, ·96059.

4. $-\tan\frac{\alpha+\beta}{2}$, $\dfrac{\cos\frac{\alpha-\beta}{2}}{\cos\frac{\alpha+\beta}{2}}$

5. $a = \frac{2}{3}\sqrt{2q^2+2r^2-p^2}$,
$\cos A = (5p^2-q^2-r^2) \div 2\sqrt{(2p^2+2q^2-r^2)(2p^2+2r^2-q^2)}$,
and similar expressions. The error in the numerator of $\cos A$ is $18x$.

8. (3, 2), (−3, −2), (2, 3), (−2, −3).

9. $3x = 4y$.

10. $x^2+y^2-18x-8y = 3$.

VII. Mechanics.

1. $P\sqrt{2}$ acting at 45° to horizontal.

2. 220 lbs. It will rest under the action of the friction alone.

3. *P* has any value from 6 to infinity.

4. $\tan^{-1}\frac{3}{10}$, $7\frac{1}{2}$ ft.

5. Former, $9\frac{1}{8}$ ft.

6. $6\frac{1}{4}$ seconds.

7. (i.) 8 oz.; (ii.) $8\frac{1}{2}$ oz.

8. 5 and 15 f.s., 900 oz.-wt.

9. 2 oz.-wt. towards centre.

10. $23\frac{1}{3}$ secs., $5288\frac{8}{9}$ ft. up the plane.

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ANSWERS.

JUNE, 1896.

II. Arithmetic.

1. $13\frac{1}{14}$. 2. 100·37. 3. $7\frac{1}{2}$ years.
4. 2 fur. 18 poles 4·4 yds. 5. $\frac{7}{40}$. 6. £1657. 16*s*. $6\frac{1}{4}$*d*.
7. 6105. 8. £4. 17*s*. 6*d*. 9. £5.
10. 3 qrs. 23 lbs. 11. £176, £320, £432. 12. 8 ft.
13. 10 cwt. zinc, 4 cwt. copper. 14. 15 miles an hour.

III. Algebra.

1. $367a - 114b + 690c$, 1082. 2. $x^6 - y^6$.
3. $-(b-c)(c-a)(a-b)(a+b+c)$, $(x-1)(x+1)(x+2)(x+3)$.
4. H.C.F. $x^2 - 10x + 21$, L.C.M. $(x^2 - 10x + 21)(x-1)(x-2)(x-4)(x-5)$.
5. 1300. 6. (i.) 1, 5; (ii.) 1, 1, 3, $\frac{1}{3}$; (iii.) $9x^2 - 27x + 20 = 0$.
7. 3. 9. $66^3 = 287496$. 10. 4368. 11. 14, 6770 ....
12. (i.) $\frac{3}{4}$; (ii.) Conservative 12944, Liberal 11120; (iii.) 21056 each.

IV. Trigonometry.

1. 3. 3. $2\cos A$, $-\cos A \pm \sqrt{3}\sin A$.
8. 90° and 15° 15′ 36″. 9. 1026·337 ft.
11. (i.) $3\frac{19}{375}$ cub. inches; (ii.) 15·65 acres.

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V. Statics and Dynamics.

1. 11 lbs.

5. $\frac{1}{3}W$. $\sqrt{\frac{10}{3}}W$.

6. Divides it in the ratio 2 : 5.

7. 96.

8. N.E. or N.W.

9. 64 ft.; time of ascent or descent 2 secs. $\frac{1}{2}$ and $3\frac{1}{2}$ secs.

10. 2 lbs.-weight.

11. $v^2 : gr$.

VI. Pure Mathematics.

1. 2 ft. 11·2 inches; 2500 : 2401.

3. 9000.

6. $x^2+x-1=0$.

8. $l\cos(\alpha-\beta)=p\pm a$.

9. $\frac{ar'+a'r}{r'-r}$, 0, and $\frac{ar'-a'r}{r'+r}$, 0. $2(a'+a)x=r'^2-r^2-a'^2+a^2$.

10. Write $-\sin 2\theta$ for $\sin 2\theta$.

11. $\frac{\sqrt{3}}{4\sqrt{2}}$, $-\frac{\sqrt{3}}{2\sqrt{2}}$.

13. A parabola with same axis and vertex at the focus of the given parabola.

14. A circle centre on the minor axis distant $CS \cot \alpha$ from the centre and radius $a \operatorname{cosec} \alpha$, where α is the given angle.

VII. Mechanics.

2. 3·38, 2·19 oz.-weight.

3. Tension $\frac{7}{40}W$, Pressures $\frac{9}{8}W$, $\frac{57}{40}W$.

4. ·634 AB.

5. $\mu^3-\mu^2-3\mu-1 : \mu^3+\mu^2-3\mu+1$.

6. It begins $220\frac{1}{2}$ ft. from the start.

7. Tension $6\frac{6}{13}$, Pressures $2\frac{10}{13}$, $2\frac{2}{13}$ oz.-weight.

8. $72(x-a)+23y=0$ and $89y=72(x+a)$.

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NOVEMBER, 1896.

II. Arithmetic.

1. $13\frac{1}{5}$.
2. 28920.
3. 3570, $\frac{1}{5}$.
4. 10*s.* 7*d.*
5. £24. 9*s.* $6\frac{5}{16}$*d.*
6. 96 yds., 29 yds. 1 ft. $\frac{32}{55}$.
7. $3\frac{1}{3}$%.
8. $33\frac{1}{2}$ ft.
9. *A* £7. 10*s.*; *B* £6. 15*s.*; *C* £10. 10*s.*
11. 2464 cubic ft.
12. 7 : 5.
13. £41. 13*s.* 4*d.*
14. 18 and 10 miles a day.

III. Algebra.

1. $(x^2+y)(x^4-x^2y+y^2)$. $\frac{1}{16}$.
2. $(1+x)(1+x^2)(1-x-x^2-x^3+x^4)$. $3x^2-5x+7$.
3. $\dfrac{x^2-xy+y^2}{x^3-x^2y+xy^2-y^3}$.
4. (i.) $6\frac{1}{4}$, $2\frac{7}{9}$; (ii.) $x=\pm\frac{1}{2}$, $\pm\dfrac{4}{\sqrt{47}}$; $y=\mp\frac{1}{3}$, $\pm\dfrac{3}{\sqrt{47}}$.
5. 105 lbs.
6. 8.
8. 520.
10. 23·0258.

IV. Trigonometry.

2. (i.) 60°, 240°; (ii.) 270°; (iii.) 0°, 180°.
3. $\sin 105^\circ=\dfrac{\sqrt{3}+1}{2\sqrt{2}}$, $\cos 105=\dfrac{1-\sqrt{3}}{2\sqrt{2}}$, etc.
4. $\frac{3}{4}$, $\frac{9}{16}$, $\frac{1}{8}$; 41° 25′.
5. *B* 107° 57′ 49″, *C* 15° 2′ 11″.
10. 4 . radius.

V. Statics and Dynamics.

2. *AB*.
3. (i.) 10 lbs.; (ii.) 15 lbs.
5. *W*.
6. Power is (i.) $\dfrac{W}{2}$, (ii.) $\dfrac{W}{\sqrt{2}}$.
7. 2 : 1.
8. (i.) *W*; (ii.) $\frac{4}{3}W$.
10. 12 ft., 4 ft.
11. Its distances from *AB*, *AC* are 16, 32 ft., and it is moving in a direction equally inclined to *AB*, *AC*.

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VI. Pure Mathematics.

2. $\frac{1}{60}(n+5)(n+10)$, $-\frac{3}{60}(n+4)(n+9)$, $\frac{3}{60}(n+3)(n+8)$, $-(n+2)(n+7)$, 0 according as n is of the form $5r$, $5r+1$, $5r+2$, $5r+3$, $5r+4$.

7. 2743·86 ft.

8. Taking centre of pentagon as origin, and a line joining it to one angular point as the axis of x, the equations are

$$x \cos 36^\circ \pm y \sin 36^\circ = \frac{a}{2} \cot 36^\circ, \quad x \sin 18^\circ \pm y \cos 18^\circ + \frac{a}{2} \cot 36^\circ = 0,$$

$$x + \frac{a}{2} \cot 36^\circ = 0, \quad 4(x^2+y^2) = a^2 \cot^2 36^\circ.$$

9. $\frac{\pi}{2} - a$, $\dfrac{n^2 \cot a}{l^2 - 2lm \operatorname{cosec} a + m^2}$.

11. A concentric ellipse semi-axes $\dfrac{ab}{\sqrt{a^2+b^2}}$, b.

VII. Mechanics.

1. $2k \,.\, AD$.

2. 38·78 and 17·22 lbs.

4. $7\frac{21}{99}$ lbs.

6. 40 miles an hour, $\frac{1}{24}$ f.s.s.

7. 4 f.s.s., 9 inches.

8. $\dfrac{4mn^2\pi^2 r}{g}$, $76\frac{4}{11}$.

9. After 1″ (32, 64); 2″ (64, 96); 3″ (96, 96); 4″ (128, 64); 5″ (160, 0); focus (80, 84).

10. $\frac{13}{64}U$, $\frac{15}{64}U$, $\frac{9}{16}U$.

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ANSWERS.

JUNE, 1897.

II. Arithmetic.

1. £670. 8*s*. $0\frac{3}{4}$*d*. 2. ·0051. 3. 437. 4. $\frac{5}{15}$.
5. $\frac{4}{15}$. 6. 14·552. 7. £688. 10*s*. 8*d*. 9. 70 days.
10. £20. 11. 25*s*. 12. 7 : 5. 13. $49\frac{93}{343}$.
14. £600, £594, £609, £621, £596, £646.

III. Algebra.

1. $-(yz+xz+xy)$, $(1-b)(1-a)(1-ab)(1+a+ab)$.
2. $6x+13$, 9709, 23287, 73. 3. 1.
4. (i.) $-\frac{4a}{3}$; (ii.) $x=y=z=-\frac{1}{3}$; $x=\frac{1}{3}$, $y=z=-\frac{2}{3}$; $y=\frac{1}{3}$, $x=z=-\frac{2}{3}$; and $z=\frac{1}{3}$, $x=y=-\frac{2}{3}$.
5. In the larger, wine : water $=p^2-32p+1280 : 32p-p^2$, and in smaller, $32-p^2 : p^2-32p+320$. $p=16$.
6. 1296 and 2025, 2025 and 3025, 3025 and 4356, etc., $13^3=91^2-78^2$.
8. (i.) $2^{30}-31931$; (ii.) $2^{29}-27841$. 10. 6·943.

IV. Trigonometry.

1. 3·1416. 4. $n\pi+(-1)^n\frac{\pi}{6}$. 6. 500. 7. 567·138 yds.
8. 54·53516 sq. in. 9. 0, -1.
10. (i.) 1232 cub. ft.; (ii.) $\sqrt{6} : \sqrt{\pi}$.

V. Statics and Dynamics.

1. 11 lbs. 15 oz. and 11 lbs. at the ends; 11 lbs. 5 oz. and 11 lbs. 10 oz. at the points of trisection.
2. 6 oz. 3. 85 lbs. 5. 4 in. 7. 45 miles an hour, or 66 f.s.
8. 3 secs., 96 f.s., 72 ft.-lbs. 9. 4 secs., 64 f.s.
10. 2 secs. 11. 16 ft., $64\sqrt{3}$ ft.

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VI. Pure Mathematics.

2. O is the orthocentre of the triangle.

3. (i.) when n is a positive integer; (ii.) when x lies between 1 and -1, when $x=1$ and $n>-1$, and when $x=-1$ and $n>0$.

4. $\frac{1}{2}(b\pm\sqrt{32a^2-b^2})$, b must not be $>4\sqrt{2}\,.\,a$.

6. $\frac{1}{2}(\operatorname{cosec} x-\operatorname{cosec} 3^n\,.\,x)$. 7. 1889·63 ft.

8. $cx+(2b-c)y=bc$, $(b-c)x-(b+c)y=0$, $bx+(b-2c)y=bc$, $(c-2b)x+cy+b^2=0$, $(b+c)x+(b-c)y=bc$, $(2c-b)x+by=c^2$, where $b=AB$, $c=AC$.

9. A straight line.

10. $\dfrac{4a^2b^2}{(a^2+b^2)^{\frac{3}{2}}}$, where a, b are the lengths of the tangents.

11. $\dfrac{b^2}{\sqrt{(a^2\cos^2\phi+b^2\sin^2\phi)}}$.

VII. Mechanics.

1. (i.) $\dfrac{4r^2+h^2}{4(r^2+h^2)}\,.\,W$; (ii.) $\frac{1}{4}W$.

2. $Q+\frac{1}{2}(\sqrt{5}+1)(P-R)$, $P+\frac{1}{2}(\sqrt{5}+1)(Q-R)$.

3. $\dfrac{b-a}{6(b+a)}\,.\,h$. 4. 13·66 lbs. 6. $37\frac{1}{7}$ ft. per hour.

8. $\tan^{-1}\frac{3}{4}$, $\tan^{-1}\frac{63}{16}$.

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NOVEMBER, 1897.

II. Arithmetic.

1. $\frac{44}{135}$. 2. ·000008. 3. £17. 10*s.*, £9. 14*s.* $5\frac{1}{3}$*d.* 4. $\frac{16}{165}$.
5. £46. 17*s.* 6*d.* 6. $3\frac{1}{3}$. 7. 162104 fr. $16\frac{2}{3}$ c.
9. £197. 17*s.* 4*d.* 10. 3 ft. $2\frac{2}{5}$ in. 11. 9·049 in.
12. *A*, 84*s.*; *B*, 72*s.*; *C*, 70*s.* 13. 15 : 14.
14. 88 yds.; 22 miles an hour.

III. Algebra.

1. $x = a^3 - 2a$, $y = 3a^2 - 1$, product $a^6 - 13a^4 + 10a^2 - 1$.
2. (i.) $a(x^2 + mx + m^2) + b(x + m) + c$; (ii.) $3xyz$;
   (iii.) $3abc(b-c)(c-a)(a-b)$.
3. (i.) $(x^2 + 1{\cdot}414x + 1)(x^2 - 1{\cdot}414x + 1)$, (ii.) $(2x + {\cdot}628)(x + 3{\cdot}186)$.
4. (i.) $a^{-\frac{1}{3}}b^{-\frac{1}{2}}$; (ii.) 4.
5. (i.) $3\frac{1}{4}$; (ii.) $\pm a(la^2 + mb^2 + nc^2)^{-\frac{1}{2}}$, $\pm b(la^2 + mb^2 + nc^2)^{-\frac{1}{2}}$,
   $\pm c(la^2 + mb^2 + nc^2)^{-\frac{1}{2}}$.
8. 76. 9. $\frac{\underline{|20}}{32\underline{|6}}$, $\frac{13\underline{|14}}{32}$.

IV. Trigonometry.

1. $4\sqrt{5}$, $2\sqrt{5}$, $8\sqrt{5}$. 2. (i.) $-$, $-$; (ii.) $+$, $+$.
3. $\frac{\sqrt{2}-1}{\sqrt{6}}$, $-\frac{\sqrt{2}+1}{\sqrt{6}}$. 4. $(2n+1)\pi$, $2n\pi \pm \frac{\pi}{3}$.
6. 3·90505 ft. 7. 23·094 ft. 9. $\frac{\pi}{4}$.
10. $\sqrt{1 + \sin\frac{a}{2}} : \sqrt{2} - \sqrt{1 + \sin\frac{a}{2}}$.

V. Statics and Dynamics.

2. $\frac{1}{3}W$, $\frac{2}{3}W$.
3. The stresses at *B* and *C* act in line *BC*, at *A* in line joining *A* to intersection of *BC* and *PQ*.
5. $W + 7w = 8P$. 6. 352, 704 yards. 7. 2 f.s.s.
8. 144 ft., 3 secs., 2 and 4 secs. 10. 4 f.s.

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VI. Pure Mathematics.

3. £30.

4. $496\frac{19}{28}$ miles, $\frac{17}{18}$ knot.

5. 2·30258, ·43429.

7. 2·9 knots.

8. A circle, with the perpendicular from the focus on the tangent as diameter.

9. $(c\cos A - b)x + c\sin A \,.\, y = 0$, A being taken as origin and AB as axis of x.

10. A circle.

11. Taking as axes AB and the perpendicular from P on AB, the centre is $(c, 0)$, and the asymptotes are $x = c$, $y = 0$.

12. (i.) 2; (ii.) 3.

VII. Mechanics.

1. 9·6 lbs. making 37° with AB.

3. (i.) $\dfrac{2ab(4a^2 - b^2)}{(4a^2 + b^2)^2} \,.\, W$; (ii.) $\dfrac{16a^4 + b^4}{(4a^2 + b^2)^2} \,.\, W$; (iii.) $\dfrac{2ab}{4a^2 + b^2} \,.\, W$.

6. 330 ft.-tons.

7. 7 : 18.

9. $\tan^{-1}\dfrac{k_2 + ek_1}{h_2 + eh_1}$ with AB.

10. $\dfrac{4}{\pi\sqrt[4]{5}} = ·85$; $\frac{3}{5}\sqrt{5}$, $\frac{9}{5}\sqrt{5}$ lbs.-wt.

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ANSWERS.

JUNE, 1898.

II. ARITHMETIC.

1. $\frac{31}{48}$. 2. 42 sq. yds. 3. 3388 sq. yds. 4. $\cdot 085\dot{2}\dot{2}\dot{7}$, $\cdot 0075$.

5. $\cdot 00161$. 6. £28. 1*s*. 7. 16380, 455 yds. 8. $\cdot 1\dot{3}$.

9. 6 cwt. 3 qrs. 41 lbs. 10. A, £680; B, £510; C, £340.

11. 168. 12. 53, 59, 61, 67, 71, 73, 79, 83, 89, 97.

13. 7 : 3. 14. 37.

III. ALGEBRA.

2. $x-3$. 3. (i.) 0; (ii.) $a+b+c$; (iii.) $\frac{4a^2b^2}{a^6-b^6}$.

4. $a^{\frac{2}{3}}x^{\frac{7}{12}}(by)^{-\frac{3}{4}}$. 5. $(q-1)^2x^2-(q^2-1)dx+qd^2=0$.

6. (i.) $\frac{(b-c)^2}{8a(b+c)}$; (ii.) 1, 1; (iii.) $-5\frac{5}{12}$, 1.

8. $\frac{\lfloor 10}{16}=226800$; $4\lfloor 7=20160$; 323, of which 72 will be repetitions in amount, but paid by different coins.

9. 2^n, $\frac{\lfloor 2n}{(\lfloor n)^2}$.

10. When n is divisible by 4 the coefficient $=-\frac{3}{n}$, otherwise $\frac{1}{n}$.

IV. TRIGONOMETRY.

1. Long hand, 510° or $\frac{17\pi}{6}$; small hand, $42\frac{1}{2}$° or $\frac{17\pi}{72}$.

2. 0°, 30°, 90°, 150°, 180°. 3. $2n\pi+\tan^{-1}\frac{b}{a}$.

6. 117″. 7. 2·72875. 9. 114·062 yds.

10. (i.) $\frac{l}{6}(ab+bc+cd)$; (ii.) $\frac{l^2}{6}(a+c+2b+2d)$, where a, b, c, d are the distances of the points where the plane cuts the horizontal edges from the corners of the cube, and l is the length of the edge of the cube.

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V. Statics and Dynamics.

1. 60°. **2.** $Q = 2w$, $AD = \frac{1}{4}AB$. **3.** 1 : 18. **4.** 7·68 inches.

5. The weights are equally distant from the horizontal line through the centre. The greatest weight $= \frac{wR}{r}$.

7. Acceleration = 8.

8. Acceleration, 2 f.s.s.; tension, $15\frac{15}{16}$ oz.; pressures, $14\frac{1}{16}$, $13\frac{13}{16}$ oz.

9. After 4 seconds, it strikes the ground 512 ft. from the tower at an angle of 45°.

10. 12 f.s.

VI. Pure Mathematics.

4. $\frac{128}{27}$. **5.** 28 years. **6.** 53° 47′ 88″ E. of N. **7.** 4.

8. 2, 2; $x+y=4$, $x-y=0$.

11. $x+a+b=0$, $4a$, $4b$ being the latera recta.

14. $\frac{(a^4y^2+b^4x^2)^{\frac{3}{2}}}{a^4b^4}$. The radius of curvature.

VII. Mechanics.

1. $\frac{1}{2}$ lb., 1·118 lbs., 45°. **4.** Angle of inclination of rod $= \tan^{-1}\mu$.

6. Unaltered. **8.** The particle projected in direction AT'.

10. Whole work done by the man $= Rh$.

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NOVEMBER, 1898.

I. Part I. First Paper.

1. $7\frac{1}{9}$. 2. £2. 18*s*. $10\frac{1}{2}$*d*. 3. $4\frac{1}{4}$ %. 4. 300 lbs.

5. 11, 24, 39. 6. $125\frac{7}{12}$. 8. $x+1$.

9. (i.) $\dfrac{a^6+1}{a(a^2-a^4-2)}$; (ii.) 3, 1; (iii.) $2\frac{1}{2}$, $1\frac{14}{29}$.

10. $\dfrac{(n-1)p-(m-1)q}{n-m}$, $\dfrac{q-p}{n-m}$; first term, 11; common diff., 3; or first term, $-\frac{37}{49}$; common diff., $\frac{3}{49}$.

11. 34. 12. 5·0003199.

II. Part I. Second Paper.

8. 324 sq. yds. 10. 972 cub. ft.

III. Part I. Third Paper.

1. $22\frac{1}{2}°$. 2. $\operatorname{cosec}^{-1}\frac{5}{4}$.

3. $-300°$, $-210°$, $-120°$, $-30°$, $60°$, $150°$, $240°$, and $330°$.

4. $1+2\cos A$. 6. $120°$.

7. ·8987253, $\bar{3}$·0370280, $\bar{1}$·9811057. 8. 30.

IV. Part II. First Paper.

1. (i.) $x=\pm 2$, $\pm\dfrac{5}{\sqrt{3}}$; $y=\pm 3$, $\pm\dfrac{1}{\sqrt{3}}$; (ii.) $x=\pm 3$, $y=2$; $x=2$, $y=\pm 3$.

2. $-\dfrac{(n-1)(2n-1)(3n-1)\ldots\overline{(m-2\,n-1)}}{\underline{|m-1}}\cdot\left(\dfrac{x}{n}\right)^{m-1}$.

3. $n\pi-\frac{1}{2}(\alpha+\beta)$. 4. 3·0505245 sq. ft.

6. The locus is a straight line parallel to BC.

V. Part II. Second Paper.

1. 16, 20 lbs. 2. $AD=4·59$ in., $AF=3·915$ in.

4. $7\frac{1}{2}$ inches from the 11 lbs. weight.

5. $5\sqrt{3}=8·66$ lbs., $5\sqrt{7}=13·22$ lbs. 6. $45°$.

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7. $180° - \cos^{-1}\frac{u}{v}$ to the direction of motion of the carriage. **8.** $\frac{2u}{f}$.

9. $\frac{8}{3}W$ lbs.-wt. vertically upwards; when moving with uniform velocity force $= W$ lbs.-wt.

10. 11 seconds.

11. (*a*) $2v$ horizontal, $\frac{v^2}{r}$ to the centre; (*b*) 0, $\frac{v^2}{r}$ to the centre.

12. $a^2 < 4h(b+h)$.

VI. Part III. First Paper.

3. The asymptotes are parallel to AB, AC bisecting the perpendiculars from O on AB, AC.

4. $(a_2b_3 - a_3b_2)(b_1x - a_1y) = a_1(a_2c_3 - a_3c_2) + b_1(b_2c_3 - b_3c_2)$ and two similar equations.

6. $\frac{b(1 \pm e\cos\phi)}{\sqrt{1 - e^2\cos^2\phi}}$. **10.** 12·9 seconds.

VII. Part III. Second Paper.

4. $2x - y = 0$. **7.** $\frac{a^2 - b^2}{a} \cdot \frac{l(1 - m^2)}{l^2 + m^2}$, $\frac{b^2 - a^2}{b} \cdot \frac{m(1 - l^2)}{l^2 + m^2}$.

8. Tension at $A = \frac{1}{3}$ wt. of board $+ \frac{Wp}{c \sin B}$, where $W =$ wt. of sphere, and p the perpendicular from its point of contact to side BC.

10. $\frac{4m - m'}{4(m + m')} \cdot u$, $\frac{5m}{4(m + m')} \cdot u$.

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ANSWERS.

JUNE, 1899.

I. Part I. First Paper.

1. $4\frac{2}{21}$. 2. 8*s.* 5*d.*, $2\frac{7}{8}$ %. 3. $208\frac{2}{7}$, 5 ft. 5·03 in.
4. 24 min. 7. $x(x+3)(x-3)(x^2+3x+9)$. 8. $14\frac{3}{4}$, 11 and $-17\frac{1}{3}$.
9. $\frac{1}{2}a-\frac{1}{6}$, $\frac{1}{2}b-\frac{1}{3}$; $100\frac{1}{3}$, $102\frac{2}{3}$.
10. 3430 is 4th term G.P., 3475 is 496th term A.P.; $\pm\frac{a}{2n+1}$.
11. 9, 15.

II. Part I. Second Paper.

1. A rectangle. 3. 2 : 1. 5. $13\frac{4}{7}$ in.
8. 9, 27 sq. inches; 3, 6, 12 and 15 sq. inches. 9. 785 cub. in.
10. $\sqrt{\left\{\frac{8A^3}{27\sqrt{3}}\right\}}$.

III. Part I. Third Paper.

1. 433,380 miles. 2. 2. 3. 70°, 160°, 250°, 340°.
5. Acute $\frac{4}{5}$, $\frac{3}{5}$, $\frac{4}{3}$; obtuse $\frac{4}{5}$, $-\frac{3}{5}$, $-\frac{4}{3}$. 6. $\frac{1}{\sqrt{2}}$, $\frac{60}{109}$.
7. 2 : 1, 1 : 3, 3 : 2. 8. A, 95° 9′ 37″; C, 64° 15′ 23″.
9. 77,613,700,000,000,000.
10. (i.) $ab=1$, (ii.) $ab=x$, (iii.) $a=b^2$, where a, b are the given numbers.

IV. Part II. First Paper.

1. $acx^2+2(a+c)bx+(a+c)^2=0$. 4. 9 : 2. 6. 86·4307 ft.

V. Part II. Second Paper.

1. W. 3. 2·68 inches from A. 4. 6 lbs.
6. $\sqrt{2}$: 1. 7. $V.\sqrt{3}$. 9. $1\frac{1}{3}$ lbs.
10. 53·76. 11. $\sqrt{\frac{g}{\sqrt{3}}}=4·29$ f.s. 12. $\frac{5}{7}V$.

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VI. Part III. First Paper.

3. $\dfrac{aa'+bb'}{\sqrt{(a^2+b^2)(a'^2+b'^2)}}$, $(3\pm 4\sqrt{3})x+(4\mp 3\sqrt{3})y+7\pm\sqrt{3}=0$.

4. 3. **5.** $(1+m^2)(x^2+y^2)-a(x+my)=0$. **12.** $\sqrt{gr}$.

VII. Part III. Second Paper.

2. The angle lies between 0° and 60°. **3.** $6x+7y=11$, $4\frac{1}{5}$.

4. $4x+3y=75$ or $4x+3y+25=0$.

5. cr. $(1-1)$, radius 1; the origin and the pt. $-\frac{8}{25}, \frac{6}{25}$.

6. $y=\pm(x+1)$. **7.** $(a^2-c^2)x^2+a^2y^2=a^2(a^2-c^2)$. **8.** 6.

9. If A, B, C be the corners at which the tensions 1, 2, 3 act respectively, the weight must be placed at the mid. pt. of CD, where $AD=2DB$.

10. $\frac{1}{4}W$. **11.** $\sqrt{\frac{3}{g}}$ seconds. **12.** $\frac{2}{3}\cdot\frac{m_1m_2}{m_1+m_2}\cdot g$.

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NOVEMBER, 1899.

I. Part I. First Paper.

1. $\frac{8}{105}$, 88. 2. £3. 6*s*. 8*d*. 3. £500. 4. 329, 42.
5. 20 miles. 6. 16, 10. 7. $x^2 - 12x + 35$.
8. (i.) 13; (ii.) 3, $-1\frac{1}{3}$. 9. $x = a + b$, $y = a - b$.
10. 325, 9. 11. 225. 12. 70, 4.

II. Part I. Second Paper.

3. 3 : 1. 5. 14 inches. 8. $3CQ = 5QD$; 5 : 16 and 3 : 16.
9. 18 ft., 105 cub. ft. 10. 4 sq. ft. 64 in., 1280 cub. in.

III. Part I. Third Paper.

3. $-\frac{3}{5}$, $-\frac{4}{5}$, $-\frac{4}{3}$; $(2n+1)180° - D$. 5. 4 ft. 5·7 in.
7. 20° 36′ 35″, 53° 7′ 48″, 106° 15′ 37″.
8. $13\frac{1}{4}$ miles, 13° 6′ 33″ west of north. 9. 1·500741.
10. (i.) 3·48429, (ii.) −·283211.

IV. Part II. First Paper.

1. (i.) $\dfrac{b^2 - 2ac}{a^2}$; (ii.) $\dfrac{1}{a^4}(b^4 - 4ab^2c + 4a^2bd + 2a^2c^2 - 4a^3e)$.
2. $\dfrac{\lfloor n}{\lfloor r \lfloor n-r}(3^r - 2^r)x^{n-r}$.
4. $2sm + \dfrac{m^2s^2}{\sqrt{s(s-a)(s-b)(s-c)}}$, $m^2\left\{\dfrac{s^2}{\sqrt{s(s-a)(s-b)(s-c)}} - \pi\right\}$.
8. 102 cub. inches.

V. Part II. Second Paper.

1. $3AC$, acting parallel to AC at D, where $BD = 2AD$. 2. 30°.
3. 8·33, 6·01 lbs.; 33° 40′ to horizontal.
4. 3 lbs. acting downwards, 6 ft. 5. $1\frac{1}{2}$, 5 in. 7. $-\frac{2}{3}$ c.s., $\frac{8}{3}$ c.s.s.
8. At heights 64, 96, 96, 64, 0 ft., greatest height, 100 ft.; 40 ft.
9. (i.) 15 grs.-wt.; (ii.) 60° to the downward drawn vertical.
10. 7 lbs., 32 f.s.s. 12. 3072 f.s., ·007446 f.s.s.

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VI. Part III. First Paper.

2. A hyperbola.

3. $x^2+y^2-2ax-\frac{a^2+2c^2}{c}.y+a^2=0.$

4. $2ax-2(b-c)y=a^2+bc-b^2.$

6. A parabola.

8. $2\sqrt{2}$ lbs.-wt.

9. $\frac{2\sqrt{P^2-F^2-W}}{2F}.$

VII. Part III. Second Paper.

4. $\frac{2\sqrt{h^2-ab}}{a+b}, \frac{2h}{a-b}.$

5. $\frac{1}{2}, -\frac{1}{2}; x+y=0.$

6. $b-a, \pm 2\sqrt{ab}.$

8. $\frac{l-2h}{2h}.w.$ If from the end of the ladder a line be drawn making the angle of friction with the vertical and meeting the vertical through the C.G. of the ladder in A, the line joining A to the point of support is the direction of the force.

9. $6:1, e=\frac{3}{4}.$

10. $2w, w; \frac{1}{2}r$ above the centre.

11. £825.

12. $\frac{a\{(1+r^2)(1-2r^{3n}+r^{4n})-r(1-r^{4n})\}}{(1-r)(1+r^2)(1-r^{3n})}.$

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ANSWERS.

JUNE, 1900.

I. PART I. FIRST PAPER.

1. 1·6, ·013. 2. £20, £66. 13*s*. 4*d*. 3. £18. 5*s*.
4. 1 ft. 2 in. 5. 12 per cent.
6. $4\frac{1}{2}$ and 6 kilometres per hour. 7. $14x^2-81x+22$.
8. (i.) 7; (ii.) -2, $\frac{1}{3}$. 9. $\frac{c-a}{c+a}$, $-\frac{c-a}{2(c+a)}$.
10. 3069. 11. $\frac{\lfloor 20}{\lfloor 15 \lfloor 5} = 15504$, $\frac{\lfloor 20}{(\lfloor 5)^4}$ 12. 16.

II. PART I. SECOND PAPER.

2. The mid-point. 7. Make the angle ABP = the angle C.
8. 89 inches. 9. 1326 sq. inches. 10. 93675 cu. metres.

III. PART I. THIRD PAPER.

1. 15°, 15′, 15″.
2. Sine $\pm\frac{12}{13}$, tangent $\pm\frac{12}{5}$, cotangent $\pm\frac{5}{12}$, cosecant $\pm\frac{13}{12}$, secant $-\frac{13}{5}$.
3. 52 ft. 4. $2n\pi+\frac{\pi}{6}$. 5. $X^2+2XY\cos\phi+Y^2$.
7. (i.) 27·0063 sq. ft.; (ii.) 5·10626 ft. 8. 54° 37′ 24″.
9. ·3010300, 2·3025851, ·6931471. 10. (i.) 2; (ii.) 43; (iii.) 42.

IV. PART II. FIRST PAPER.

1. $v-u$, $(n-1)v-(n-2)u$, $\frac{n}{2}\{(n-1)v-(n-3)u\}$.
2. $p(p+q)(p+2q)\ldots(p+\overline{r-2}q)x^{r-1}\div\lfloor r-1$.
6. (i.) 447·87 links; (ii.) 122·235 links.
8. 684·233 sq. inches. 9. 2.

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V. PART II. SECOND PAPER.

1. $R \sin \theta$. **2.** 34·8 lbs. **3.** $8\frac{1}{3}$ lbs.

4. $W \sin(45° \pm \alpha)$. **5.** $56\frac{1}{2}$ lbs. **6.** $\frac{\sqrt{61}}{12}$ of length of side.

7. $\frac{v^2}{2f}$. **9.** $\sqrt{\left\{2\left(g \sin \alpha - \frac{F}{m}\right) h \operatorname{cosec} \alpha\right\}}$, $\frac{mg \sin \alpha - F}{mg \sin \alpha + F} . h$ vertically.

10. (i.) $\sqrt{\frac{m - m'}{m + m'} . gh}$; (ii.) $\frac{\sqrt{(m^2 - m'^2) g . h}}{2m}$.

VI. PART III. FIRST PAPER.

3. $\frac{13}{5}$. **4.** $\frac{2}{7}\sqrt{3}$, $\frac{4}{7}$. **10.** Stable.

11. 30 miles an hour. **12.** $\frac{9\sqrt{3}}{19}$.

VII. PART III. SECOND PAPER.

3. $(\overline{m + n}k - 2mn)x + (m + n - 2k)y = 0$.

4. $x^2 + 5xy + 2y^2 - 7x - 9y + 8 = 0$; $x - y = 0$, $7x + 9y = 8$.

5. $x^2 + y^2 - 2a(x + y) + a^2 = 0$; $\frac{1}{2} \sin^{-1} \frac{1}{8}$ and $\frac{\pi}{2} - \frac{1}{2} \sin^{-1} \frac{1}{8}$.

7. $b^2 x + a^2 y \tan \theta = 0$. **9.** $\frac{m'v}{m + m'}$ at angle α with line joining mm'.

10. 10 lbs., $1\frac{1}{4}$ lbs.

11. (i.) $\sqrt{gl}$; (ii.) $\frac{1}{2}\sqrt{gl}$ vertical, $\frac{1}{2}\sqrt{5gl}$ at an angle $\tan^{-1} 2$ to the vertical.

12. A parabola.

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NOVEMBER, 1900.

I. PART I. FIRST PAPER.

1. $29\frac{3}{7}$. 2. ·28845. 3. $12\frac{9}{11}$ acres, £249. 19*s*. $1\frac{1}{11}$*d*.
4. £116. 13*s*. 6*d*. 5. 6·3, 31·5 seconds. 6. 721.
8. 0, $4\frac{3}{5}$; $c = \frac{125}{12}$. 9. $\frac{b+c-a}{a+b-c}$, $\frac{a+c-b}{a+b-c}$.
10. $a^{\frac{2}{3}} - b^{\frac{2}{3}}$.
12. $1 - \frac{1}{2}x^2 + \frac{3}{8}x^4 - \frac{5}{16}x^6 + \frac{35}{128}x^8 - \ldots \quad \frac{1}{x} - \frac{1}{2x^3} + \frac{3}{8x^5} - \frac{5}{16x^7} + \frac{35}{128x^9} - \ldots$
When $x = 10$, value = ·099504.

II. PART I. SECOND PAPER.

2. 100°. 3. 5·3, 1·7 inches. 7. The tangent from O to the circle.
8. 156 sq. ft. 9. $2r$. 10. 186 lbs.

III. PART I. THIRD PAPER.

1. 206264·8 seconds. 2. $n\pi + (-1)^n\frac{\pi}{6}$. 3. 1; 120°.
4. 45°. 5. $abc \sin a \div 2\sqrt{s(s-a)(s-b)(s-c)}$.
6. −150°, −1. 7. 81·4885 ft.
8. $c \sin A \,.\, \sqrt{a^2 - c^2 \sin^2 A}$. 9. (i.) 4; (ii.) −4. 10. 1·608.

IV. PART II. FIRST PAPER.

1. $(x+y)(y+z)(z+x)$. 2. Between −3 and −2, 0 and 1, 1 and 2.
6. 2504 ft. 8. A straight line.

V. PART II. SECOND PAPER.

3. 8·6 lbs.-wt. 4. 7 lbs.-wt., 9 lbs.-wt. 5. 4·76 lbs.-wt.
6. 69 lbs.-wt., 30 lbs. 7. 2·07 ft. per sec. 9. $\frac{1}{2}g$, $\frac{1}{3}mg$.
10. $\frac{5}{4}\sqrt{65}$ lbs.-wt. acting at an angle $\tan^{-1}8$ with the direction of motion; 9 : 7.
11. 29·3 ft. before. 12. $\sqrt{\frac{gl \sin^2 \alpha}{\cos \alpha}}$.

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VI. PART III. FIRST PAPER.

2. A hyperbola with the given points as foci.

3. $\dfrac{p\sin\beta - q\sin\alpha}{\sin(\beta-\alpha)}$, $\dfrac{q\cos\alpha - p\cos\beta}{\sin(\beta-\alpha)}$.

4. $\dfrac{[(p+q)(pr^2+qs^2)-pq\{(\alpha-\gamma)^2+(\beta-\delta)^2\}]^{\frac{1}{2}}}{p+q}$.

5. $90^\circ - \alpha$ with Ox;

$$\frac{(g\cos\alpha+f\sin\alpha)\{g\sin\alpha\cos\alpha+f(\sin^2\alpha-2)\}+c\sin\alpha}{2(f\cos\alpha-g\sin\alpha)}.$$

$$\frac{(f\sin\alpha+g\cos\alpha)\{f\sin\alpha\cos\alpha+g(\cos^2\alpha-2)\}+c\cos\alpha}{2(g\sin\alpha-f\cos\alpha)}.$$

7. (i.) $(ax^2+by^2-1)(ap^2+bq^2-1)=(apx+bqy-1)^2$;

(ii.) $4ab(ap^2+bq^2-1)=\{ab(p^2+q^2)-(a+b)^2\}\tan^2\alpha$, and $ap^2+bq^2>1$.

8. $\dfrac{Wa^2\sin\theta}{\sqrt{b^2-4a^2\sin^2\frac{\theta}{2}}}$. **10.** v, 0. **11.** 60 miles an hour.

12. 39·144 in., $85200\dfrac{h}{r}$.

VII. PART III. SECOND PAPER.

3. The st. line $3x-11y=0$.

4. $8x^2-2xy-15y^2+11y-2=0$, $8x^2-2xy-15y^2=0$.

5. $x-h+(y-k)\tan\alpha=0$.

9. (i.) $m'>m(\sin\alpha+\mu\cos\alpha)$,

(ii.) $m(\sin\alpha-\mu\cos\alpha)>m$;

accelerations $\dfrac{m'-m(\sin\alpha+\mu\cos\alpha)}{m+m'}g$, $\dfrac{m(\sin\alpha-\mu\cos\alpha)-m'}{m+m'}g$.

10. 2640 lbs.-wt., 488000 ft.-lbs.

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FEB 4 1902

DEC 6 [illegible]

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Math 395.30.21
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